A STATIC STATE ESTIMATOR FOR A POWER SYSTEM NETWORK

by

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Submitted to the Department of Electrical Engineering on August 26, 1968 in partial fulfillment of the requirements for the Degree of Master of Science.

ABSTRACT

The security of a power system can be improved by a central computer control. One important aspect of this central computer control is the processing of "noisy" observations of the state. The state is defined here as the voltage magnitudes and voltage phase angles at all the buses (nodes) of the system.

A Monte-Carlo computer simulation of a high voltage transmission system and the state estimation program is constructed to investigate the process of estimating the state from the observations. The bulk (high voltage) transmission system is modeled as a network consisting of power generators, loads, and transmission lines, and is assumed to be operating at steady state, using single phase lines, and working at one voltage level. A scaled-down version of a transmission system model is simulated on the computer by node analysis calculations and load flow calculations. The observations of the system are measurements of bus voltage magnitudies, real powers, and reactive powers. The power measurements may be taken at the buses and/or on the transmission lines; only powers at buses are used in this program. The measurements are of two types: real and psuedo. Real measurements come directly from the meters monitering the system; psuedo measurements are "best" guesses based on past data. The "noise" in the measurements is assumed to have Gaussian statistics; the psuedo measurement noise has a much larger variance than that of the real measurement noise. The estimator is simulated by means of a vector recursion equation derived from non-linear Fisher estimation theory.

The simulation program first investigates the conditions under which an estimate can be obtained. The results indicate that for a well metered system and for low noise levels a very accurate estimate can be calculated. The estimator calculates a matrix which is expected to be, and actually is, a fairly good approximation to the error covariance matrix of the estimate, provided the estimate is accurate and the noise levels are low. This matrix allows a sensitivity analysis to be performed easily and quickly.

The simulation program is also used to investigate the affects of meter placement, meter noise, and meter type on the accuracy of the estimate. A thorough analysis is not completed, but some basic results are found.

Many further studies are suggested.

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1. INTRODUCTION

The real time central control of an electrical power system's bulk generation and extra high voltage transmission system by a large digital computer is a subject which is receiving much recent attention. Roughly, the operation of the central control system is divided into two steps: 1) gathering of real time information from around the system and processing it, and 2) formulating and applying control decision based on the processed information.

This paper investigates one topic of the first step: processing the available information into an estimate of the "state" of the system. The definition of "state" depends on what is of most interest. In this paper, the "state" of the system is the column vector whose elements are the voltage magnitudes and voltage angles at all the buses (or nodes) of the system. Moreover, this paper considers only the static state; system dynamics are not included.

The estimation of the state of the power system is very important to the central control, because effective decision laws to tell the system where to go in the future depend on a knowledge of where the system is in the present. The implementation and study of the static state estimator, then, is an important step in realizing central computer control of a power system.

The static state estimator calculates the best estimate of the state based on "noisy" observations of the system, which are expressed as non-linear functions of the state. Since the functions are non-linear the estimator operates recursively to find the best estimate.

The estimator is studied by means of a Monte Carlo computer simulation using a large scale digital computer (IBM 360). The simulation program is constructed as follows. A general model of a bulk (high voltage) distribution system is simulated on the computer. From the simulated power system, true values of the state and power flows are calculated. The true measurements are taken of the system and noise is added to simulate the "noisy" measurements of the system. The estimator program, which is set up on the computer, processes the noisy measurements to obtain an estimate of the state of the system. The performance of the estimator is then analyzed by various means such as comparing the true state with the estimated state.

This paper begins by presenting a general model for the power system. Next a theory of network analysis and the load flow calculations are presented. An example model is described and the theory of network analysis is applied to the model. The general theory of non-linear estimation is then presented. Included in this section is a discussion of the error covariance matrix and its approximations. The relationships between the load flow calculations and the estimator calculations is also discussed in detail. A computer program which simulates a power system and the static state estimator has been constructed, and the program is described in some detail.

The computer program uses the example model to see how well the static state estimator works, and to understand how the estimator performs under various conditions. The results are presented and analyzed in the last section.

NOTATION

All vectors and matrix variables are underlined. Transpose is denoted by "T". E [ ] is the expectation operator of probability theory.

Section 2. BACKGROUND

2.1. A Model for a Power System Network

A power system delivers electrical power from generating stations to loads via transmission lines. A well developed power system integrates a large number of generating stations so that their combined output is readily available throughout the region served. For example, there are many alternate pathways or loops for power to flow from the generators to the loads. A power system operates at several different voltage levels: high voltage for bulk power transmission, lower voltage for distribution of power to loads. A typical network may have hundreds of buses (nodes or tie points) connecting the transmission lines to loads and generators. Also a power system of one company is usually interconnected to adjoining companies, creating, in effect, one system over most of the continental United States and Canada.

A power system is not static. Loads, for instance, are always changing their power demands with minute by minute, daily, weekly, seasonal, and yearly variations. Also, the network is always changing due to transmission line losses or additions, or generator losses or additions. There are many other items to be considered before the power system is completely modeled. However, a basic but general model of the power system is constructed which makes several approximations and simplifications of the real system, so that the analyses will be easier. These approximations are the following:

1. The model operates at steady state with a frequency of 60 cycles per second.
2. The network of the system is completely known, and can be completely monitored.
3. The three phase lines are balanced. Hence, single line representation is used.

4. The lower voltage distribution system is radial and so is included in the load.

A real power system, as mentioned previously, is never in steady state. However, for short intervals of time, such as five minutes or less, the system is nearly in steady state, because the changes in the system are so small. It is not unrealistic, then, to consider the system to change from one steady state operating point to another over intervals of about five minutes.

COMPONENTS

The model consist of three basic elements: power generators, loads, and transmission lines. These elements are joined together at the buses. (Note that there is no difference between the work "bus" and the word "node"). One bus, the ground or datum bus, serves as a reference for voltage measurements and is usually not included in the diagram of the model.

The power generator is modeled as an ideal constant power source which delivers constant power for any voltage across the generator. The load is similarly modeled as a constant power sink which draws constant power for any voltage across the load. The transmission line can be represented in various ways depending on its length, voltage level, etc. Transmission line theory is used to find the lumped parameter model of the line. For instance, the lumped parameter model of a medium length, high voltage line is found to be a pi network consisting of a resistance and inductance in series and two shunt capacitors to ground. (See Figure 2.1) An example model used in this paper is described in Appendix A.

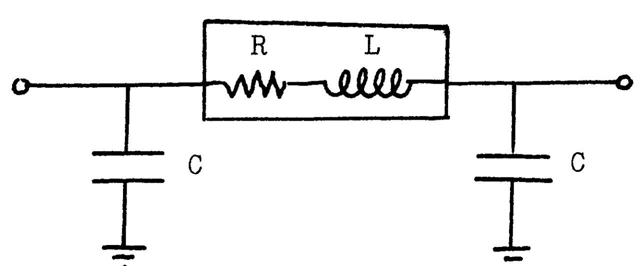

Fig. 2.1
Transmission Line

2.2. Network Analysis

The power system, which is now modeled as a network made up of lumped parameter elements, generator, and loads, is analyzed by the node analysis method to obtain the equations for the bus voltages and the powers flowing along the lines and into the buses.

Assume that the power system has, in general, n buses and b branches. Define the following scalar and vector quantities:

1. $\underline{v}_i$ (i=1, b) = complex branch voltages

and

$$\underline{v}_i = \begin{bmatrix} v_1 \\ \cdot \\ \cdot \\ \cdot \\ v_b \end{bmatrix}$$ = complex branch voltage vector.

2. i_i (i=1, b) = complex branch current in branch i

$$\underline{I} = \begin{bmatrix} i_1 \\ \cdot \\ \cdot \\ \cdot \\ i_b \end{bmatrix}$$ = complex branch current vector.

3. j_i (i=1, b) = complex branch source current in branch i

$$\underline{J} = \begin{bmatrix} j_1 \\ \cdot \\ \cdot \\ \cdot \\ j_b \end{bmatrix}$$ = complex branch source current vector

4. y_i (i=1, b) = complex branch admittance of branch i.

$$\underline{Y}_{branch} = \begin{bmatrix} y_1 & & & \underline{0} \\ & \ddots & & \\ & & y_i & \\ \underline{0} & & & \ddots \; y_b \end{bmatrix}$$ = complex branch admittance matrix.

5. E_i $(i = 1, n)$ = complex bus voltage measured from bus i to ground

In polar form $$E_i = |E_i| e^{j\delta_i}$$

where $|E_i|$ is the bus voltage magnitude and δ_i is the bus voltage phase angle.

$$\underline{E} = \begin{bmatrix} E_1 \\ \cdot \\ \cdot \\ \cdot \\ E_n \end{bmatrix}$$ = complex bus voltage vector

6. $$\underline{x} = \begin{bmatrix} \delta_i \\ \cdot \\ \cdot \\ \cdot \\ \delta_n \\ |E_1| \\ \cdot \\ \cdot \\ \cdot \\ |E_n| \end{bmatrix}$$ = state vector of system.

VOLTAGE EQUATIONS

The elemental branch is shown in Figure 2.2.

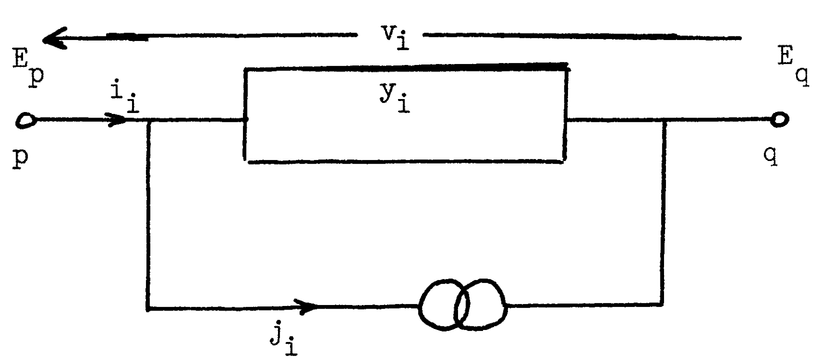

Fig. 2.2

Elemental Branch

From Ohm's law:

$$\underline{i}_i = \underline{j}_i + \underline{y}_i \cdot \underline{v}_i \tag{2.2-1}$$

or in vector form

$$\underline{I} = \underline{J} + \underline{Y}_{branch} \underline{V} \tag{2.2-2}$$

Also

$$\underline{v}_i = \underline{E}_p - \underline{E}_q \tag{2.2-3}$$

Now define the node incidence matrix $\underline{A}$ where the elements of the matrix are:

$$a_{ij} = \begin{cases} 1, & \text{if the } j^{th} \text{ branch is incident to and issues from the } i^{th} \text{ bus.} \\ -1, & \text{if the } j^{th} \text{ branch is incident to and enters into the } i^{th} \text{ bus.} \\ 0, & \text{if the } j^{th} \text{ branch is not incident to the } i^{th} \text{ bus.} \end{cases}$$

From the definition of the node incidence matrix $\underline{A}$, it can be seen that:

1. Kirchoff's Current Law:

$$\underline{A} \cdot \underline{I} = 0 \tag{2.2-4}$$

2. Kirchoff's Voltage Law:

$$\underline{V} = \underline{A}^T \cdot \underline{E} \tag{2.2-5}$$

To calculate the bus voltages, equation (2.2-2) is premultiplied by $\underline{A}$.

$$\underline{A} \cdot \underline{I} = \underline{A} \cdot \underline{J} + \underline{A} \cdot \underline{Y}_{branch} \cdot \underline{V} \tag{2.2-6}$$

Substituting equations (2.2-4) and (2.2-5) into (2.2-6) it follows that

$$-\underline{A}\ \underline{J} = \left[\underline{A}\ \underline{Y}_{branch}\ \underline{A}^T\right] \cdot \underline{E} \tag{2.2-7}$$

Define the following vector and matrix quantities:

$$\underline{J}_{bus} = \underline{A} \cdot \underline{J}\ , \tag{2.2-8}$$

where $\underline{J}_{bus}$ = complex bus source current vector.

$$\underline{Y}_{bus} = \underline{A}\ \underline{Y}_{branch}\ \underline{A}^T\ , \tag{2.2-9}$$

where $\underline{Y}_{bus}$ = complex bus admittance matrix.

Hence the bus equation is:

$$-\underline{J}_{bus} = \underline{Y}_{bus} \cdot \underline{E} \qquad (2.2\text{-}10)$$

Studies in node analysis show that $\underline{Y}_{bus}$ can be easily determined by inspecting the network.

$$(\underline{Y}_{bus})_{ij} \Big|_{i = j} = \text{the sum of } (\underline{Y}_{branch})_k \text{ , k = all branches}$$

connected to bus i.

$$(\underline{Y}_{bus})_{ij} \Big|_{i \neq j} = - \text{the sum of } (\underline{Y}_{branch})_k \text{ , k = all branches}$$

which connect buses i and j.

The bus equations are also written in polar form. Define the following polar coordinate relations:

$$\underline{E}_i = |\underline{E}_i| \ e^{j \delta i}$$

where $|E|_i$ is the bus voltage magnitude and δ_i is the bus voltage angle.

$$(\underline{Y}_{bus})_{i_j} = |\underline{Y}|_{i_j} \cdot e^{-j \theta i_j}$$

where $|\underline{Y}|_{i_j}$ is the bus admittance magnitude and θ_{i_j} is the bus admittance angle.

The equations then become:

$$-(\underline{J}_{bus})_i = \sum_{j=1}^{n} |\underline{E}_j \cdot \underline{Y}_{ij}| \ e^{-j (\theta_{ij} - \delta_j)} \qquad (2.2\text{-}11)$$

$(i = 1, n)$

<u>POWER EQUATIONS</u>

Define now the complex power at bus i to be:

$$\underline{P}_i - j \underline{Q}_i = - \underline{E}_i^* \cdot (\underline{J}_{bus})_i \qquad (2.2\text{-}12)$$

where $\underline{E}_i^*$ represents the complex conjugate of $\underline{E}_i$, $\underline{P}_i$ is real power flowing into the network from bus i, $\underline{Q}_i$ is reactive power flowing into the network from bus 1. Writing the power equations in polar form:

$$\underline{P}_i - j\,\underline{Q}_i = \sum_{j=1}^{n} |\underline{E}_i \cdot \underline{E}_j \cdot \underline{Y}_{ij}|\; e^{-j\,(\theta_{ij} + \delta_i - \delta_j)} \tag{2.2-13}$$

Separating real and imaginary parts gives

$$\underline{P}_i = \sum_{j=1}^{n} |\underline{E}_i \cdot \underline{E}_j \cdot \underline{Y}_{ij}|\; \cos\,(\theta_{ij} + \delta_i - \delta_j) \tag{2.2-14a}$$

$$\underline{Q}_i = \sum_{j=1}^{n} |\underline{E}_i \cdot \underline{E}_j \cdot \underline{Y}_{ij}|\; \sin\,(\theta_{ij} + \delta_i - \delta_j). \tag{2.2-14b}$$

$(i=1, n)$

The equations written in polar form are more useful for analysis and will be used from now on.

In addition, the power flows along the lines are calculated. Make the following definitions concerning the transmission line impedances. (See Fig. 2.3)

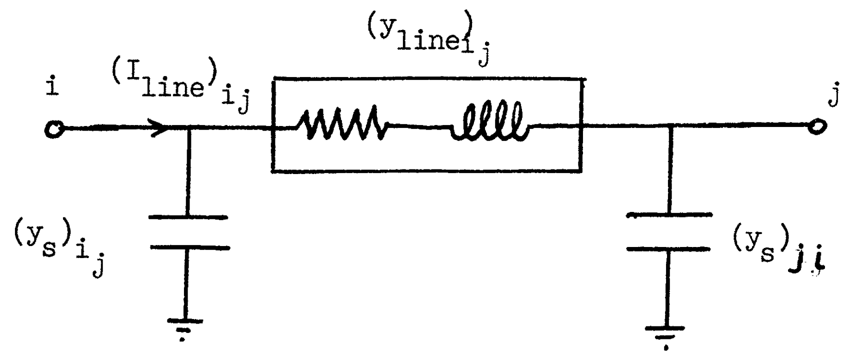

Fig. 2.3.

Transmission Line Admittances

where,

$(y_\ell)_{ij}$ = line admittance of the transmission line connecting buses i and j.

$(y_s)_{ij}$ = shunt admittance at bus i of the line connecting buses i and j.

In polar coordinates,

$$(y_\ell)_{ij} = |y_\ell|_{ij} \, e^{-j\,(\theta_\ell)_{ij}}$$

Also

$$(y_s)_{ij} = |y_s|_{ij} \quad e^{-j(\theta_s)_{ij}}$$

Usually,

$$(y_s)_{ij} = (y_s)_{ji}$$

Also,

$(I_{line})_{ij}$ = line current from bus i into the transmission line connecting buses i and j.

From Ohm's law it is found that

$$(-I_{line})_{ij} = (E_i - E_j) \cdot (y_\ell)_{ij} + E_i \cdot (y_s)_{ij} \tag{2.2-15}$$

In polar form

$$-(I_{line})_{ij} = E_i (y_\ell)_{ij} e^{-j((\theta_\ell)_{ij} - \delta_i)} - E_j (y_\ell)_{ij} e^{-j((\theta_\ell)_{ij} - \delta_j)}$$

$$+ E_i (y_s)_{ij} e^{-j((\theta_s)_{ij} - \delta_i)} \tag{2.2-16}$$

The power at bus i flowing into the transmission line connecting buses i and j is defined as

$$(P_{line})_{ij} - j(Q_{line})_{ij} = -E_i^* \cdot (I_{line})_{ij} \tag{2.2-17}$$

where $(P_{line})_{ij}$ is the real power, and $(Q_{line})_{ij}$ is the reactive power.

In polar coordinates,

$$(P_{line})_{ij} - j(Q_{line})_{ij} = |E_i \cdot E_i \cdot (y_\ell)_{ij}| e^{-j(\theta_\ell)_{ij}}$$

$$- |E_i E_j (y_\ell)_{ij}| e^{-j((\theta_\ell)_{ij} + \delta_i - \delta_j)}$$

$$+ |E_i E_i (y_s)_{ij}| e^{-j((\theta_s)_{ij})}$$

(2.2-18)

Separating real and reactive parts,

$$(P_{line})_{ij} = |E_i E_i (y_\ell)_{ij}| \cos((\theta_\ell)_{ij})$$

$$- |E_i E_j (y_\ell)_{ij}| \cos((\theta_\ell)_{ij} + \delta_i - \delta_j)$$

$$+ |E_i E_i (y_s)_{ij}| \cos((\theta_s)_{ij}) \tag{2.2-19}$$

$$(Q_{line})_{ij} = |E_i\, E_i\, (y_\ell)_{ij}|\ \sin\left((\theta_\ell)_{ij}\right)$$

$$-|E_i\, E_j\, (y_\ell)_{ij}|\ \sin\left((\theta_\ell)_{ij} + \delta_i - \delta_j\right)$$

$$+ |E_i\, E_i\, (y_s)_{ij}|\ \sin\left((\theta_s)_{ij}\right) \qquad (2.2\text{-}20)$$

Note that

$$P_i - jQ_i = \sum_{j=1} (P_{line})_{ij} - j\,(Q_{line})_{ij}\ ,$$

j = all buses connected to bus i

and that the power in the transmission line connecting buses i and j equals the algebraic sum of the powers flowing into the transmission line.

There are many references on this subject. Two references used are Desoer and Kuh (1), and Stagg and El-Abiad (2).

2.3. Load Flow Calculations

Load flow calculations provide power flows and voltages for a given power system to specific load requirements, generating capabilities and voltage specifications. Among other things these calculations provide essential information for the evaluation of the present and future performance of a power system.

At each bus there are four quantities: real and reactive power, voltage magnitude, and voltage phase angle. At each bus, two of the four quantities are specified. It is necessary to select one bus, called the slack bus, to provide the additional real and reactive power to supply the transmission line losses, since these are unknown until the final solution is obtained. At this bus, the voltage magnitude and phase angle are specified. At the remaining buses real and reactive powers are specified. Some systems have voltage controlled buses, and for these buses the voltage magnitude and real power are specified. However, in this paper, no voltage controlled buses are used; nevertheless, it is an interesting point which will be mentioned later when discussing the relation of the estimator to load flow calculations.

As before assume there are n buses with bus n as the slack bus. The load flow calculations solve the following 2n-2 non-linear equations for the voltage magnitudes and phase angles:

$$\underline{P}_i = \sum_{j=1}^{n}{}' \left| \underline{E}_i \cdot \underline{E}_j \cdot \underline{Y}_{i_j} \right| \cos (\theta_{i_j} + \delta_i - \delta_j) \qquad (2.3\text{-}1)$$

$$\underline{Q}_i = \sum_{j=1}^{n}{}' \left| \underline{E}_i \cdot \underline{E}_j \cdot \underline{Y}_{i_j} \right| \sin (\theta_{i_j} + \delta_i - \delta_j) \qquad (i = 1, n\text{-}1)$$

where $\underline{P}_i$ and $\underline{Q}_i$ are the real and reactive powers defined in (2.2-12).

These equations can be solved in several ways. In this paper, the Newton-Raphson method is used, because it clarifies the similarities and differences between the load flow calculations and the estimator calculations.

The power equations can be written in vector form. Let

$$\underline{z} = \begin{bmatrix} P_1 \\ \cdot \\ \cdot \\ \cdot \\ P_{n-1} \\ Q_1 \\ \cdot \\ \cdot \\ \cdot \\ Q_{n-1} \end{bmatrix} = \text{power vector}$$

and

$$\underline{x} = \begin{bmatrix} \delta_1 \\ \cdot \\ \cdot \\ \cdot \\ \delta_{n-1} \\ |E_1| \\ \cdot \\ \cdot \\ \cdot \\ |E_{n-1}| \end{bmatrix} = \text{state vector with } \delta_n \text{ and } |E_n| \text{ removed,}$$

and

$$\underline{H}(\underline{x}) = \begin{bmatrix} \underline{H}_1(\underline{x}) \\ \cdot \\ \cdot \\ \cdot \\ \underline{H}_{2n-2}(\underline{x}) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} |\underline{E}_i \, \underline{E}_j \, \underline{Y}_{ij}| \cos(\theta_{ij} + \delta_i - \delta_j) \\ \cdot \\ \cdot \\ \cdot \\ \sum_{j=1}^{n} |\underline{E}_i \, \underline{E}_j \, \underline{Y}_{ij}| \sin(\theta_{ij} + \delta_i - \delta_j) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

so that

$$\underline{z} = \underline{H}\ (\underline{x}) \qquad (2.3\text{-}2)$$

is the vector form of the non-linear equations to be solved.

The Newton-Raphson method linearizes the non-linear equations by a Taylor series expansion around an initial guess of the state.

$$\underline{z} = \underline{z}_o + \frac{\partial \underline{H}}{\partial \underline{x}}\ (\underline{x})_o \cdot \Delta\ \underline{x} + \ldots\ldots \qquad (2.2\text{-}3)$$

where

$$\Delta \underline{x} = \underline{x}_o - \underline{x}_o$$

and

$$\underline{x}_o = \text{the initial guess}$$

$$\underline{z}_o = \underline{H}\ (\underline{x}_o)$$

$$\frac{\partial \underline{H}}{\partial \underline{x}} = \text{Jacobian matrix of partial derivatives}$$

$$= \begin{bmatrix} \frac{\partial \underline{H}_1}{\partial \underline{x}_1} & \cdots & \frac{\partial \underline{H}_1}{\partial \underline{x}_n} \\ \vdots & & \vdots \\ \frac{\partial \underline{H}_{2(n-2)}}{\partial \underline{x}_1} & \cdots & \frac{\partial \underline{H}_{2(n-2)}}{\partial \underline{x}_n} \end{bmatrix}$$

$$= \underline{H}_x\ (\underline{x}_o)$$

Ignoring higher order terms in the expansion results in a linear equation for $\Delta \underline{x}$.

$$\Delta \underline{x} = \underline{x} - \underline{x}_o$$

$$= \underline{H}_x^{-1} (\underline{x}_o) \quad (\underline{z} - \underline{z}_o) \tag{2.3-4}$$

An iteration scheme is constructed: Let the next initial guess of the state be the sum of the old initial guess and the correction term $\Delta \underline{x}$.

$$\underline{x}_1 = \underline{x}_o + \Delta \underline{x}$$

or in general,

$$\underline{x}_{i+1} = \underline{x}_i + \Delta \underline{x}_i \tag{2.3-5}$$

where,

$$\Delta \underline{x}_i = \underline{H}_x^{-1} (\underline{x}_i) \; (\underline{z} - \underline{z}_i) \tag{2.3-6}$$

The interation is stopped when $\underline{H}(\underline{x}_i)$ is sufficiently close to the true values of the real and reactive bus powers.

From the calculated bus voltages the power at the slack bus is calculated. The calculated voltages and power flows are the "true" values of the system, and will henceforth be referred to as the true voltage magnitude and true phase angle (E_t , δ_t) and true powers (P_t , Q_t).

The equations for the load flow calculations written in full are:

$$\begin{bmatrix} \Delta \underline{P}_i \\ \\ \Delta \underline{Q}_i \end{bmatrix} = \begin{bmatrix} \underline{P}_{sked} - \underline{P}_i \\ \\ \underline{Q}_{sked} - \underline{Q}_i \end{bmatrix}$$

$$= \begin{bmatrix} \underline{J}_1 & \underline{J}_2 \\ \underline{J}_3 & \underline{J}_4 \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_i \\ \Delta |E|_i \end{bmatrix} \tag{2.3-7}$$

for i = 1, n-1.

The terms $\underline{P}_{sked}$ and $\underline{Q}_{sked}$ are the scheduled bus powers at n-1 buses, and $\underline{J}_1$, $\underline{J}_2$, $\underline{J}_3$, $\underline{J}_4$ are matrices whose elements are

$$\underline{J}_1 = \begin{cases} \left.\dfrac{\partial P_i}{\partial \delta_j}\right|_{i \neq j} = |Y_{ij}\, E_i\, E_j| \sin(\theta_{ij} + \delta_i - \delta_j) \\ \dfrac{\partial P_i}{\partial \delta_i} = -\sum\limits_{\substack{j=1 \\ i \neq j}}^{n} |Y_{ij}\, E_i\, E_j| \sin(\theta_{ij} + \delta_i - \delta_j) \end{cases} \qquad (2.3\text{-}8)$$

$$\underline{J}_2 = \begin{cases} \left.\dfrac{\partial P_i}{\partial |E|_j}\right|_{i \neq j} = |E_i\, Y_{ij}| \cos(\theta_{ij} + \delta_i - \delta_j) \\ \dfrac{\partial P_i}{\partial |E|_i} = 2|E_i\, Y_{ii}| \cos(\theta_{ii}) \\ \qquad + \sum\limits_{\substack{j=1 \\ i \neq j}}^{n} |E_j \cdot Y_{ij}| \cos(\theta_{ij} + \delta_i - \delta_j) \end{cases} \qquad (2.3\text{-}9)$$

$$\underline{J}_3 = \begin{cases} \left.\dfrac{\partial Q_i}{\partial \delta_j}\right|_{i \neq j} = -|E_i \cdot E_j \cdot Y_{ij}| \cos(\theta_{ij} + \delta_i - \delta_j) \\ \dfrac{\partial Q_i}{\partial \delta_i} = \sum\limits_{\substack{j=1 \\ i \neq j}}^{n} |E_i \cdot E_j \cdot Y_{ij}| \cos(\theta_{ij} + \delta_i - \delta_j) \end{cases} \qquad (2.3\text{-}10)$$

$$\underline{J}_4 = \begin{cases} \left.\dfrac{\partial Q_i}{\partial |E|_j}\right|_{i \neq j} = |E_i \cdot Y_{ij}| \sin(\theta_{ij} + \delta_i - \delta_j) \\ \dfrac{\partial Q_i}{\partial |E|_i} = 2|E_i \cdot Y_{ii}| \sin(\theta_{ii}) \\ \qquad + \sum\limits_{\substack{j=1 \\ i \neq j}}^{n} |E_j \cdot Y_{ij}| \sin(\theta_{ij} + \delta_i - \delta_j) \end{cases} \qquad (2.3\text{-}11)$$

The recursion equation is

$$\begin{bmatrix} \underline{\delta}_{i+1} \\ |\underline{E}|_{i+1} \end{bmatrix} = \begin{bmatrix} \underline{J}_1 & \underline{J}_2 \\ \underline{J}_3 & \underline{J}_4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta \underline{P}_i \\ \Delta \underline{Q}_i \end{bmatrix} + \begin{bmatrix} \underline{\delta}_i \\ |\underline{E}_i| \end{bmatrix} \qquad (2.3\text{-}12)$$

for i=1, n-1.

The results of the load flow calculations for the example model are given in Appendix B. For more information on load flow calculations and for other references on this topic see Stagg and El-Abiad (2).

Section 3. ESTIMATION

The central control of the power system needs to know the present state and present power flows of the system from real time measurements of the system. The static state estimator supplies this information by calculating a "best" estimate of the state from the "noisy" measurements. It should be noted here that the estimator calculations are not the same as the load flow calculations, because the estimator employs the statistics of the noise in the calculations to "weight" the measurements, and because it can use more measurements than unknowns. More will be said about this topic later.

3.1. Metering

A power system has many types of meters that monitor system in real time. Three types of meters frequently used are:

1. wattmeters which measure real power flow
2. varmeters which measure reactive power flow
3. voltmeters which measure magnitude of voltage.

Other types of meters include ammeter, power factor meters, etc. These meters are placed on the buses and on the transmission lines to measure the bus voltages, generator powers, load powers, and powers flowing in the transmission lines.

The measurements are sent to the central control via microwave channels or telephone lines where they are processed.

Let the vector $\underline{z}_{real}$ denote the set of real time measurements sent to the computer. Then

$$\underline{z}_{real} = \underline{z}_T + \underline{v}_{real} \qquad (3.1\text{-}1)$$

where $\underline{z}_T$ = the meter readings assuming no errors are made

$\underline{v}_{real}$ = the errors of the real measurement which come from many sources, such as meter noise, communication noise, the effects of analog to digital conversion, and modeling errors in assuming balanced three phase lines.

The measurements are not all direct, real time, observations of the system. Another type of measurement, called a psuedo-measurement, is also included. Psuedo-measurements are guesses of the actual value of a variable, such as power flow or voltage magnitude, based on past data or design specifications. This data comes in various forms such as percentages of nominal or scheduled values, or direct measurements, etc. Let $\underline{z}_{psuedo}$ represent the set of psuedo measurement sent to the computer. Then

$$\underline{z}_{psuedo} = \underline{z}_T + \underline{v}_{psuedo} \tag{3.1-2}$$

where $\underline{z}_T$ = the calculated values of the variables not measured directly.

$\underline{v}_{psuedo}$ = the errors associated with the psuedo measurements.

The two types of measurements, real and psuedo, can be combined into one vector equation known as the observation equation. Let $\underline{z}$ represent all the measurements or observations of the system sent to the computer.

Then,

$$\underline{z} = \begin{bmatrix} \underline{z}_{real} \\ \underline{z}_{psuedo} \end{bmatrix} = \underline{z}_T + \underline{v} \tag{3.1-3}$$

where $\underline{z}$ = the complete set of measurements assuming no errors, $\underline{v}$ = the complete set of measurement errors.

The error vector $\underline{v}$ is modeled as a zero mean random vector with

$$E\ (\underline{v}\ .\ \underline{v}^T)\ = \underline{R}, \qquad (3.1\text{-}4)$$

where $\underline{R}$ is the positive definite error covariance matrix. The statistics of the psuedo-measurement errors are usually determined from the past data as the average of the squares of the deviations of actual values determined using past data from the nominal values. The variances of the psuedo measurements are usually larger than the direct (real) measurement errors.

Frequently, the noise is assumed to have Gaussian statistics, because nothing else can be found which works better and because the mathematics are easier to deal with. For these reasons, the gaussian assumption is used in this paper.

Let $\underline{x}$ represent the state of the system. If the network is completely known, then

$$\underline{z}_T = \underline{H}\ (\underline{x}), \qquad (3.1\text{-}5)$$

where $\underline{H}\ (\underline{x})$ is a set of non-linear functions of $\underline{x}$ determined from Kirchoff's laws and Ohm's law.

3.2. Estimation Theory.

The best estimate of the state, represented by $\hat{\underline{x}}$, is to be determined from the observation equation (3.1-3). The statistics of $\underline{x}$ are completely unknown, so that Fisher estimation is used. See Appendix C for details on Fisher estimation theory. If Gaussian statistics are assumed, the Fisher estimate of the state $\hat{\underline{x}}$ is that value of $\underline{x}$ which minimizes the quadratic function J where

$$J = (\ \underline{z} - \underline{H}\ (\underline{x})\ )^T \cdot \underline{R}^{-1} \cdot (\ \underline{z} - \underline{H}\ (\underline{x})\ ). \qquad (3.2\text{-}1)$$

However, $\underline{H}(\underline{x})$ is a set of non-linear functions, and so an iteration scheme is used to minimize J. Let $\underline{x}_o$ be an initial guess of the state. Expand the observation equation about $\underline{x}_o$

$$\underline{z} = \underline{H}(\underline{x}_o) + \frac{\partial \underline{H}}{\partial \underline{x}} (\underline{x}_o) \Delta \underline{x} + \ . \ . \ . + \underline{v} \ , \qquad (3.2\text{-}2)$$

where $\Delta \underline{x} = \underline{x} - \underline{x}_o$.

and $\dfrac{\partial H}{\partial x} = \underline{H}_x$ = matrix of partial derivatives.

If higher order terms in $\Delta \underline{x}$ are ignored, then

$$\Delta \underline{z} = \frac{\partial \underline{H}}{\partial \underline{x}} (\underline{x}_o) \cdot \Delta \underline{x} + \underline{v} \qquad (3.2\text{-}3)$$

where $\Delta \underline{z} = \underline{z} - \underline{H}(\underline{x}_o)$

This observation equation is linear, and so linear Fisher estimation theory can be applied.

Let $\Delta \hat{\underline{x}}$ denote the best estimate of change in state calculated from Fisher theory. Then

$$\Delta \hat{\underline{x}} = \left[\underline{H}x^T (x_o) \ \underline{R}^{-1} \ \underline{H}x (\underline{x}_o) \right]^{-1} \underline{H}x^T (\underline{x}_o) \ \underline{R}^{-1} \left[\underline{z} - \underline{H}(\underline{x}_o) \right]$$

(3.2-4)

A new initial guess of the state is found by adding the old initial guess $\underline{x}_o$ to the estimate of the change in the state, $\Delta \hat{\underline{x}}$. The calculations are then repeated.

In general, the interation procedure is:

$$\underline{\hat{x}}_{i+1} = \underline{\hat{x}}_i + \Delta \underline{\hat{x}}_i \qquad (3.2\text{-}5)$$

where

$$\Delta \underline{\hat{x}}_i = \left[\underline{H}_x^T (\underline{\hat{x}}_i) \cdot \underline{R}^{-1} \cdot \underline{H}_x (\underline{\hat{x}}_i)\right]^{-1} \cdot \underline{H}_x^T (\underline{\hat{x}}_i) \cdot \underline{R}^{-1} \cdot \left[\underline{z} - \underline{H}(\underline{\hat{x}}_i)\right] \qquad (3.2\text{-}6)$$

and where $\underline{\hat{x}}_i$ is the i^{th} estimate of the state.

3.3. An Example of the Observation Equation

For illustrative purposes an example of the observation equation is written out. Assume the network has n buses, and that there are ne voltmeters, nq varmeters and np wattmeters at specified buses; there are no wattmeters or varmeters measuring power flow along transmission lines.

The observation equation then is

$$\begin{bmatrix} |\underline{E}_i| \\ \vdots \\ \underline{P}_k \\ \vdots \\ \underline{Q}_\ell \end{bmatrix} = \begin{bmatrix} |\underline{E}|_i \\ \vdots \\ \sum_{j=1}^{n} |Y_{k_j} \cdot E_k \cdot E_j| \cos(\theta_{k_j} + \delta_k - \delta_j) \\ \vdots \\ \sum_{j=1}^{n} |Y_{\ell_j} E_\ell E_j| \sin(\theta_{\ell_j} + \delta_\ell - \delta_j) \end{bmatrix} \qquad (3.3\text{-}1)$$

where i = buses where voltmeters are placed (ne buses)

k = buses where wattmeters are placed (np buses)

ℓ = buses where varmeters are placed. (nq buses)

and $|\underline{E}_i|$ = column vector of voltage magnitudes

$\underline{P}_k$ = column vector of real powers

$\underline{Q}_\ell$ = column vector of reactive powers

The linearized observation equation is

$$\begin{bmatrix} \Delta|E_i| \\ \Delta \underline{P}_k \\ \Delta \underline{Q}_\ell \end{bmatrix} = \begin{bmatrix} \underline{0} & \left(\dfrac{\partial|E_i|}{\partial|E_q|}\right) \\ \underline{J}_1' & \underline{J}_2' \\ \underline{J}_3' & \underline{J}_4' \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_p \\ \\ \Delta |E|_q \end{bmatrix} \qquad (3.3\text{-}2)$$

where $p = 1, n - 1$

$q = 1, n$

and $\underline{J}_1' = \dfrac{\partial \underline{P}_k}{\partial \underline{\delta}_p}$ (dimension np x (n-1))

$\underline{J}_2' = \dfrac{\partial \underline{P}_k}{\partial |\underline{E}|_q}$ (dimension np x n)

$\underline{J}_3' = \dfrac{\partial \underline{Q}_\ell}{\partial \underline{\delta}_p}$ (dimension nq x (n-1))

$\underline{J}_4' = \dfrac{\partial \underline{Q}_\ell}{\partial |\underline{E}|_q}$. (dimension (nq x n))

It should be noticed that there are n voltage magnitudes but only n-1 voltage phase angles in the state vector. This is because only the differences in angles are important, and so the angles are not all independent. One angle is used as a reference, in this case it is the angle at bus n.

The zero matrix comes from $\dfrac{\partial |E|_i}{\partial \delta_p}$, and has a dimension of ne x (n-1).

The matrix $\dfrac{\partial |E|_i}{\partial |E|_q}$ has a dimension of ne x n.

3.4. Error Covariance Matrix

In the appendix on the Fisher linear estimation theory, (Appendix C), it is mentioned that the error covariance matrix is equal to the following matrix:

$$E\left[(\underline{x}_T - \hat{\underline{x}})(\underline{x}_T - \hat{\underline{x}})^T\right] = \left[\underline{H}^T \ \underline{R}^{-1} \ \underline{H}\right]^{-1} \tag{3.4-1}$$

where $\underline{x}_T$ of the true state, x is the estimated state and

$$\underline{z} = \underline{H} \cdot \underline{x} + \underline{v}$$

is the linear observation equation. This is not true in general for a non-linear Fisher estimator. However, under the conditions that the noise error is small, and that the estimated state is unbiased and sufficiently close to the true state, then a good approximation to the error covariance matrix is:

$$E\left[(\underline{x}_T - \underline{x}) \cdot (\underline{x}_T - \underline{x})^T\right] = \sum(\hat{x}) \tag{3.4-2}$$

where

$$\sum(\hat{\underline{x}}) = \left[\underline{H}_x^T \ (\hat{\underline{x}}) \cdot \underline{R}^{-1} \cdot \underline{H}_x \ (\hat{\underline{x}})\right]^{-1} \tag{3.4-3}$$

The following arguments attempt to explain the conditions under which this is true.

Recall the estimator recursion equations (3.2-5) and (3.2-6):

$$\hat{\underline{x}}_{i+1} = \hat{\underline{x}}_i + \sum(\hat{\underline{x}}_i) \cdot \underline{H}_x^T \ (\hat{\underline{x}}_i) \cdot \underline{R}^{-1} \cdot \left[\underline{z} - \underline{H} \ (\underline{x}_i)\right] \tag{3.4-4}$$

where

$$\sum(\hat{\underline{x}}_i) = \left[\underline{H}_x \ (\hat{\underline{x}}_i) \cdot \underline{R}^{-1} \cdot \underline{H}_x \ (\hat{\underline{x}}_i)\right]^{-1}$$

Let $\underline{x}$ be the "best" estimate of the state. The non-linear Fisher estimate is, in general, biased; that is,

$$E\ (\hat{\underline{x}}) = \underline{x}_T + b\ (\underline{x}_T)\ , \tag{3.4-5}$$

where $b\ (\underline{x}_T)$ is some bias function of the true state, $\underline{x}_T$.

Let

$$\hat{\underline{x}}_{i+1} = \underline{G}(\underline{z}) = \Sigma(\hat{\underline{x}}_i)\ \underline{H}_x^T\ (\hat{\underline{x}}_i)\ \underline{R}^{-1} \left[\underline{z} - \underline{H}(\underline{x}_i)\right] + \hat{\underline{x}}_i \tag{3.4-6}$$

represent the estimate as a function of the observations. Then,

$$E\ (\underline{G}\ (\underline{z})\ ) = E\ (\hat{\underline{x}})$$

$$= (\underline{x}_T + \underline{b}\ (\underline{x}_T))$$

substituting the non-linear observation equation (3.1-3) into (3.4-6)

$$\hat{\underline{x}} = \underline{G}\ (\underline{H}\ (\underline{x}_T)\ ) + \underline{v})$$

Expanding $\underline{G}$ in a Taylor series about $\underline{H}\ (\underline{x}_T)$ gives,

$$\underline{x} = \underline{G}\left[\underline{H}\ (\underline{x}_T)\right] + \underline{G}^{(1)}\left[\underline{H}(\underline{x}_T)\right]\underline{v} + \ldots \tag{3.4-7}$$

where

$$(\underline{G}^{(1)}\ (\underline{z}) = \frac{\partial \underline{G}(\underline{z})}{\partial \underline{z}}\ )$$

the matrix of partial derivatives of $\underline{G}$.

Assuming $\underline{v}$ is small, so that the higher order terms in the expansion can be neglected, then

$$\underline{G}\left[\underline{H}(\underline{x}_T)\right] = \underline{x}_T + \underline{b}\ (\underline{x}_T) \tag{3.4-8}$$

Hence,

$$\hat{\underline{x}} - \underline{E}(\hat{\underline{x}}) = \underline{G}^{(1)}\left[\ \underline{H}(\underline{x}_T)\right]\ \underline{v}$$

$$\hat{\underline{x}} - \underline{x}_T - b(\underline{x}_T) = \underline{G}^{(1)}\left[\underline{H}(\underline{x}_T)\right].\ \underline{v} \tag{3.4-9}$$

Since

$$E\left[\underline{v} \cdot \underline{v}^T\right] = \underline{R} ,$$

then

$$E\left[(\underline{x} - \underline{x}_T) \cdot (\underline{x} - \underline{x}_T)^T\right] = \underline{G}^{(1)}\left[\underline{H}(\underline{x}_T)\right]\underline{R}\left[\underline{G}^{(1)}\left[\underline{H}(\underline{x}_T)\right]\right]^T$$

$$+ \underline{b}(\underline{x}_T) \cdot \underline{b}^T (\underline{x}_T) \qquad (3.4\text{-}10)$$

The first derivative of $\underline{G}$ can be obtained from equation (3.4-6). Then

$$\underline{G}^{(1)} (\underline{z}) = \sum(\underline{x}) \cdot \underline{H}_x^T (\hat{\underline{x}}) \cdot \underline{R}^{-1} \qquad (3.4\text{-}11)$$

Hence,

$$\underline{G}^{(1)}\left[\underline{H}(\underline{x}_T)\right] \cdot \underline{R}\left[\underline{G}^{(1)}\left[\underline{H}(\underline{x}_T)\right]\right]^T = (\Sigma \underline{H}_x^T \; \underline{R}^{-1}) \cdot \underline{R} \cdot (\underline{R}^{-1} \underline{H}_x \Sigma)$$

$$= \sum(\hat{\underline{x}}) \qquad (3.4\text{-}12)$$

If the estimated state is close to the true state, and if the bias error $\underline{b}(\underline{x}_T)$ is small enough to be ignored, then

$$\underline{E}\left[(\hat{\underline{x}} - \underline{x}_T)(\hat{\underline{x}} - \underline{x}_T)^T\right] = \underline{\sum}(\hat{\underline{x}})$$

$$= \left[\underline{H}_x^T (\hat{\underline{x}}) \cdot \underline{R}^{-1} \cdot \underline{H}_x (\hat{\underline{x}})\right]^{-1} \qquad (3.4\text{-}13)$$

Since the bias error is included in (3.4-10) it is better to call this analysis a sensitivity analysis rather than an error analysis.

One of the purposes of the computer program which simulates the estimator is to find when this approximation is good.

It is hoped that the approximation is accurate, because much computer time will be saved if $\underline{\Sigma}(\hat{\underline{x}})$ can be used to approximate the error covariance matrix instead of using the matrix $\underline{S}$, where

$$\underline{S} = 1/N \sum_{i=1}^{N} \left[(\underline{x} - \underline{x}_T)_i \cdot (\underline{x} - \underline{x}_T)_i^T \right], \qquad (3.4\text{-}14)$$

to approximate the error covariance matrix.

Section 4. Similarities and Differences between Load Flow Calculations and the Estimator Calculations

The load flow calculations and the estimator calculations are not the same, because they are evolved from different logics and used for different purposes. Yet there are similarities, too. The following list outlines the relationships between the load flow calculations and the estimator calculations.

Load Flow

$$\underline{z} = \underline{H}\,(\underline{x})$$

where $\underline{z} = \begin{bmatrix} P_i \\ Q_i \end{bmatrix}$

for i = 1, n-1.

Or for voltage controlled buses

$$\underline{z} = \begin{bmatrix} P_i \\ Q_j \\ |E_k| \end{bmatrix}$$

for $i = 1, n-1$

j = all load buses

k = all voltage controlled buses

$$\underline{x} = \begin{bmatrix} \delta_i \\ |E|_i \end{bmatrix}$$

for $i = 1, n-1$

Estimator

$$\underline{z} = \underline{H}\,(\underline{x}) + \underline{v}$$

where $\underline{z} = \begin{bmatrix} |E_i| \\ P_j \\ Q_k \\ (P_{line})_{lm} \\ (Q_{line})_{pq} \end{bmatrix}$

and i, j, k etc., are the buses where $|E|$, P, Q, P_{line}, Q_{line} measurements are taken.

$$\underline{x} = \begin{bmatrix} \delta_i \\ |E|_j \end{bmatrix}$$

for $i = 1, n-1$

$j = 1, n$

Recursion formula:

$$\Delta \underline{x}_i = \underline{H}_x^{-1} \left[\underline{z} - \underline{H} (\underline{x}_i) \right] \qquad \Delta \underline{x}_{i+1} = \left[\underline{H}_x^T (\hat{\underline{x}}_i) \, \underline{R}^{-1} \, \underline{H}_x (\hat{\underline{x}}_i) \right]^{-1} \cdot \underline{H}_x^T (\hat{\underline{x}}_i) \, \underline{R}^{-1} \left[\underline{z} - \underline{H} (\hat{\underline{x}}_i) \right]$$

$$\underline{x}_{i+1} = \underline{x}_i + \Delta \underline{x}_i \qquad \hat{\underline{x}}_{i+1} = \hat{\underline{x}}_i + \Delta \hat{\underline{x}}_i$$

SIMILARITIES:

From this list the similarities are readily found. They are:

1. Both calculations make use of the real and reactive bus power equations.

2. If $\underline{H}_x (\hat{\underline{x}}_i)$ in the estimator calculations is square and invertible, the estimator recursion formula becomes

$$\Delta \hat{\underline{x}}_i = \underline{H}_x^{-1} (\hat{\underline{x}}_i) \left[\underline{z} - \underline{H} (\hat{\underline{x}}_i) \right]$$

which is very similar to the load flow recursion formula.

3. The same type of recursion formula is used. (Newton-Raphson)

4. The state vector is almost the same.

5. If no $|E|$ measurements and no line power measurements are made, then equations for load flow and the estimator are almost exactly the same.

6. With the use of voltage controlled buses, the load flow equations are even more similar to the estimator equation.

DIFFERENCES

The list also points out the differences. They are:

1. The estimator calculations include noise and the statistics of the noise in the calculations to weight the observations in proportion to their error. The load flow calculations assumes all specified values are exactly known.

2. The estimator can use more measurements than there are unknowns in the state vector. This redundancy in observations is sometimes useful for obtaining a more accurate estimate of the state.

3. The state vector for the estimator is one dimension larger, because all n voltage magnitudes are included.

4. There is no slack bus in the estimator calculation; hence, all the powers could be included in the vector $\underline{z}$.

5. The load flow calculations can use psuedo measurements in the calculations. These psuedo measurements may have large errors, yet they are still useful. The estimator program can handle the psuedo measurements, because it weights the measurements in proportion to their accuracy. Hence, the psuedo measurements are not weighted very heavily, but nevertheless, some information is obtained from them.

6. The estimator calculates an approximation to the error covariance matrix automatically. (see Section 3.4).

Section 5. Construction of Simulation Program

The flow chart in Figure 5.1 illustrates the basic set-up of the static state estimator. The set-up is know as a Monte Carlo network of simulation and can be divided into four parts: 1) the model of the power system, 2) the noise generator, metering and formulation of the observation vector, 3) the estimator, and 4) a comparison of the estimated state with the true state.

The first section simulates the model of the power system by calculating the true bus voltage magnitudes $\underline{E}_T$ and true voltage phase angles, $\underline{\delta}_T$ as well as the true real and reactive powers (P_T , Q_T) at the buses by means of load flow analysis.

The next section generates the gaussian random noise vector by means of a random number generator. Each "noise" has a zero mean and a variance specified in the input data. The observation vector is formulated by selecting the measurements to be taken and then adding the true meter readings to the "noise" vector to obtain noisy observations.

The third section is the estimator which takes the noisy observations and uses them to solve the recursion equation to obtain the estimate of the state. The estimator also calculates the real and reactive power using the estimated state, and prints out the matrix $\sum$(or just its diagonal).

The last section compares the true state with the estimate in some fashion in order to analyze the performance of the estimator. More will be said about this comparison later.

Program listings and detailed flow charts for all the programs and subprograms are given in Appendix D.

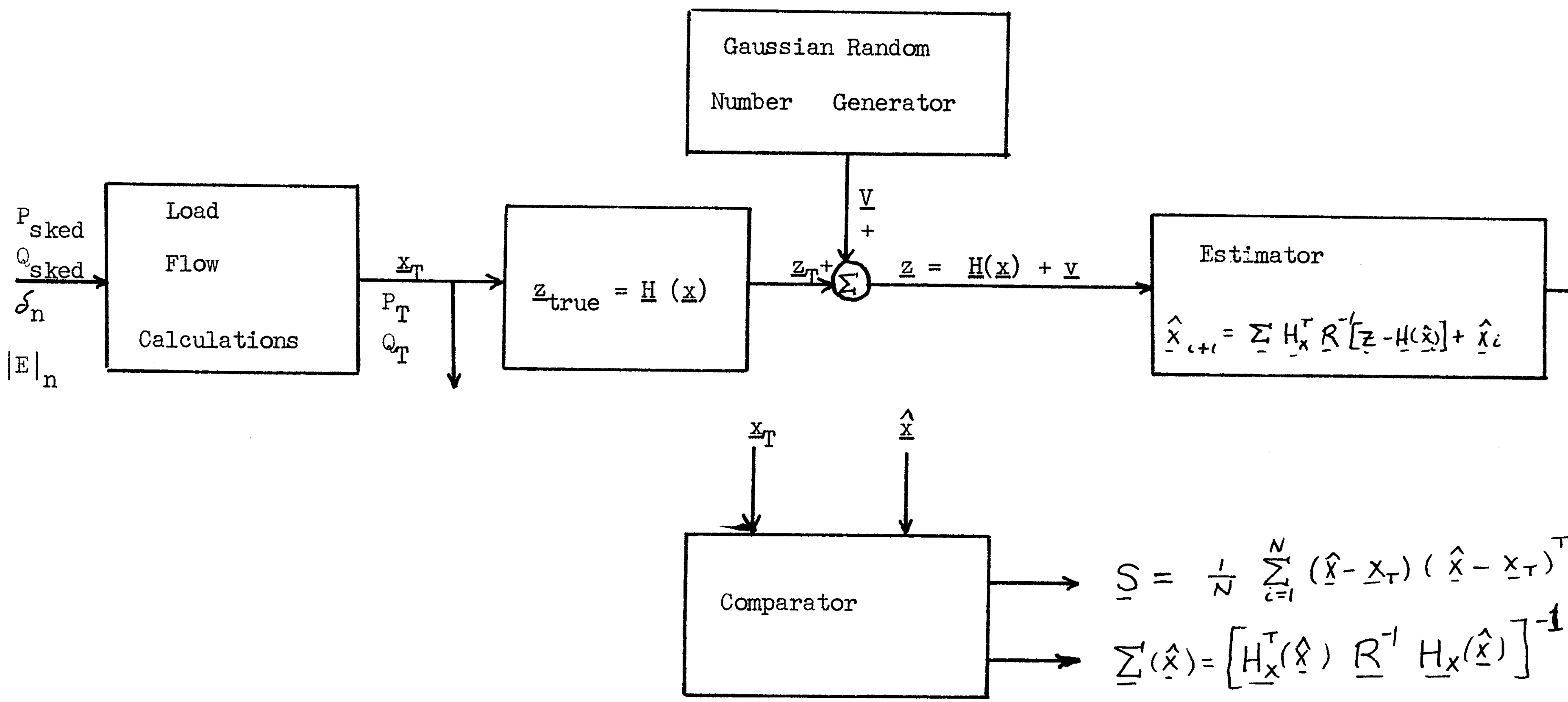

Fig. 5.1
Computer Simulation

Section 6. Analysis of the Computer Studies

The purpose of this paper is to construct a static state estimator for a power system network and to analyze the performance of the estimator. In this section, the results from the computer program simulation are presented and analyzed. The analysis is divided into two parts:

1) Checking the estimator to see if it is working properly, investigating the limitations of the estimator, and comparing $\Sigma(\hat{\underline{x}})$ to the error covariance matrix.
2) Analyzing the performance of the estimator for various types of measurements and meter placement schemes.

The estimator program is checked by answering the following questions:

1. Does the estimator program converge to a finite estimate?
2. Is the percentage error of the estimate less than or equal to the percentage of noise in the measurements?
3. Is the number of iterations to converge small (e.g. 5-10)?

In addition, the limitations of the estimator are studied. First, the measurement noise is increased to see how the error of the estimate increases. Secondly, the initial guess of the state is increased to see if this effects the convergence of the estimator. Finally, the minimum number of measurements is used to see if an estimate can be obtained. The results of these studies give some indication of the conditions under which the estimator can still perform adequately.

Another important item is to check how well the matrix $\Sigma(\hat{\underline{x}})$ approximates the error covariance matrix, which cannot be calculated exactly. Instead, it is approximated by the matrix $\underline{S}$, where

$$\underline{S} = \frac{1}{N} \sum_{i=1}^{N} (\underline{x}_T - \hat{\underline{x}})\ (\underline{x}_T - \hat{\underline{x}})^T \tag{6.1}$$

and where N is the total number of trials using different measurements, $\underline{x}_T$ is the true state, $\hat{\underline{x}}$ is the estimated state.

The second part of the analysis investigates parts of the following areas:

1. The relative importance of the type of meter in obtaining an accurate estimate.
2. The importance of meter placement in obtaining an accurate estimate.
3. The affect of measurement errors on the accuracy of the estimate.

These areas are very important and basic, but they are by no means the only areas that can be explored. There are, for instance, many interesting interrelations between these areas that can be studied.

The following approximations are also investigated:

$$\Delta \underline{P} \cong \underline{J}_1 \cdot \Delta \underline{\delta} \tag{6.2}$$

$$\Delta \underline{Q} \cong \underline{J}_4 \cdot \Delta |\underline{E}| \tag{6.3}$$

provided the reactance to resistance ratio (X/R) is large. These approximations are obtained from the power equations (2.2-14a) and (2.2-14b) by approximating sin $(\Delta \underline{\delta})$ by $\Delta \underline{\delta}$ and cos $(\Delta \underline{\delta})$ by 1.0, provided

$\Delta \underline{\delta} \ll 1^\circ$ and $(X/R) \gg 1$.

In terms of the estimator program, this means that a small change in the noise of the watt measurements does not affect the accuracy of the voltage magnitude estimate. Likewise a small change in the noise of the var measurements does not affect the accuracy of the phase angle estimates.

The example model shown in Appendix A is used for all these studies. As can be seen from the network diagram, it is a very basic model which facilitates the analysis of the computer studies. The subscripts on the voltage magnitudes and angles and on the real and reactive powers are the code numbers assigned to each bus. Also these studies use only measurements of voltage magnitudes and real and reactive powers at the buses; no power flows along lines are measured.

The example system network is simulated by the load flow calculations. The true state and the true bus powers are calculated, and the results are listed in Appendix C.

NOTE ON THE TABLES

The values of the results listed in Tables 6.1 through 6.9 are in per unit quantities. The reference base is 100 KVA at 138 KV. All angles are in per unit radians, voltages in per unit kilovolts, real power in per unit megawatts, and reactive power in per unit megavars.

6.1. Estimator Checkout

The first computer runs check out the estimator program to see if it is working properly. Some sample results are given in Table 6.1. The results indicate that the estimator calculations do converge to a finite estimate, and that for a well metered system the percent error of the estimate is less than or equal to the percent of the noise in the measurements. It is also found that for an initial starting point near the true state, the typical number of iterations is quite reasonable, especially for the strict convergence criterion used ($|\hat{x}_{i+1} - \hat{x}_i| \leq .00005$). Sometimes the number of iterations can be large(e.g. 22). However, this occurs rarely. In order to study the rate of convergence, a plot of the residual of one element of the state (in this case $\Delta|E|_5 = |E_T|_5 - |\hat{E}|_5$) versus the iteration count is drawn. The result of several samples are shown in Figure 6.1. It can be seen that for accurate observations, the estimate changes very fast in the first few steps, and then

Meter Type		E	P	Q	E	P	Q	E	P	Q	E	P	Q	E	P	Q	E	P	Q
Buses Metered		1	1		1	1		1	1	1	1	1	1		1	1	1	1	
			2	2		2	2	2	2	2	2	2	2		2	2	2	2	
		3	3	3	3	3	3	3	3	3	3	3	3		3	3	3	3	
			4	4		4	4	4	4	4	4	4	4		4	4	4	4	
		5		5	5		5	5	5	5	5	5	5		5	5	5	5	
Measurement Error (%)		1	1	1	5	5	5	1	1	1	10	10	10		1	1	1	1	
Number of Iterations	Typ.		6			8			6			5			6			6	
	Max.		8			22			10			10			6			8	
Sample Error of Estimate*	$\lvert\underline{\delta}_T-\hat{\underline{\delta}}\rvert$		.00053 .00097 .00111 .00133			.00272 .00497 .00570 .00677			.00001 .00004 .00006 .00006			.00320 .00193 .00294 .01106			.00015 .00007 .00013 .00025			.00135 .00162 .00131 .00164	
	$\lvert\lvert E\rvert_T-\lvert\hat{E}\rvert\rvert$		.00273 .00315 .00318 .00313 .00271			.01369 .01582 .01594 .01569 .01361			.00093 .00095 .00091 .00012 .00096			.01997 .01931 .01982 .02355 .01960			.00186 .00191 .00191 .00178 .00191			.00085 .01170 .00172 .01240 .00388	

*in per-unit quantities

Table 6.1.

Number of Interations to Converge

and Accuracy of Estimated State

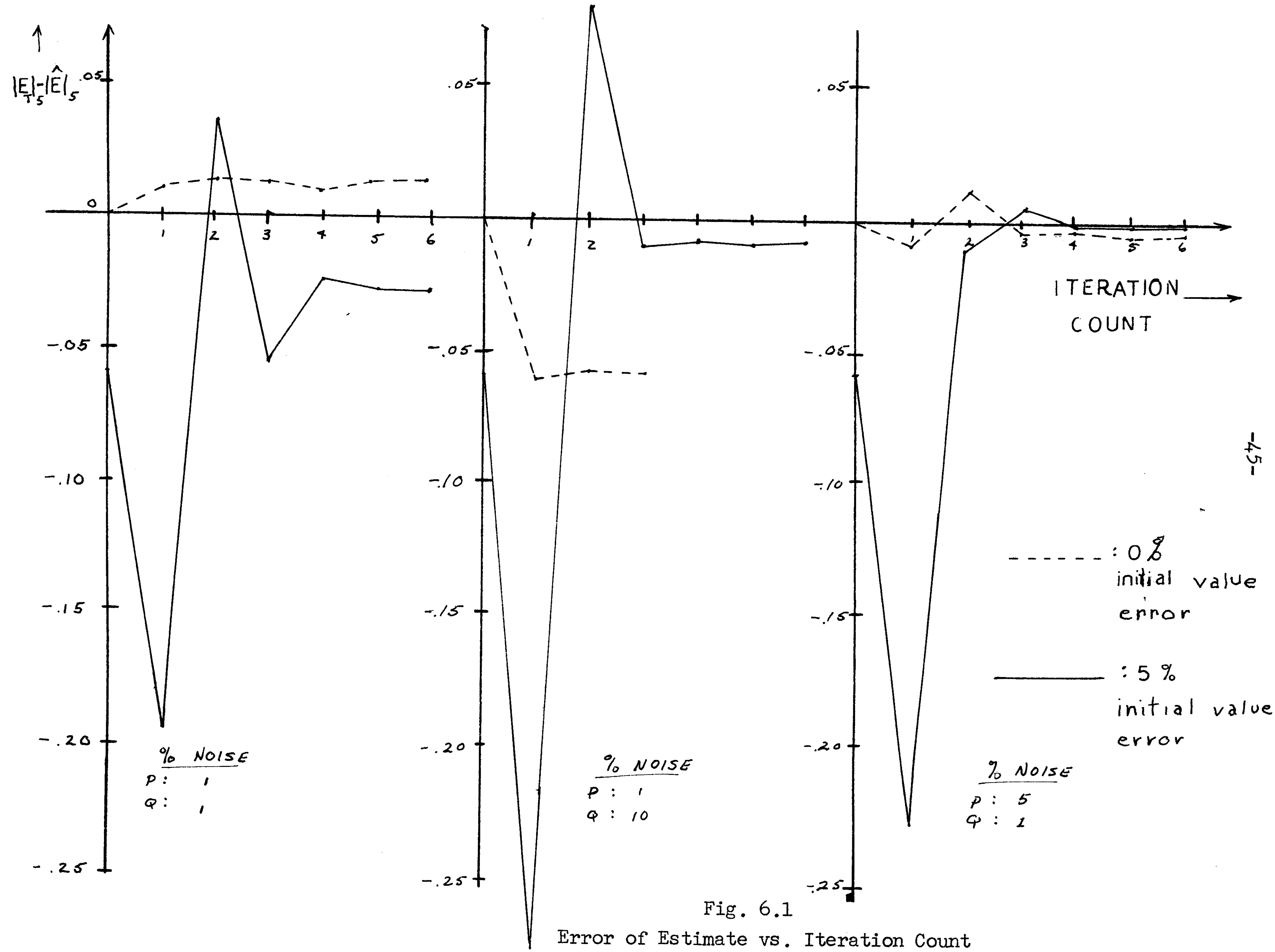

Fig. 6.1
Error of Estimate vs. Iteration Count
(No |E| METERS – P & Q METERS ONLY)

levels off when close to the stopping conditions. For higher noise levels, the estimate oscillates before leveling off. Notice that the number of iterations does not depend on the starting value. It also seems that the number of iterations does not depend on the buses metered, or the noise level, provided that the system is well metered and the noise level is reasonably low. What does seem to affect the number of iterations is the value of the measurments themselves; some sets of measurements cause the estimator to take longer to converge than other measurements. This topic needs to be investigated further.

Notice that when the measurement noise increases, the error in the estimate increases roughly in proportion, as is expected. It is also seen that the accuracy of the estimate changes significantly for certain meter placement patterns; meter placements are important.

In conclusion, for a well metered sytem and for initial values of the state close to the true state, the estimator program achieves a good estimate of the state.

6.2. Limitations of the Estimator

The estimator program is run under several extreme conditions. These conditions are large measurement noises, large errors in initial starting point, and the use of only the minimum number of buses. The results are listed in Table 6.2.

LARGE MEASUREMENT NOISE

In order to exclude errors due to meter placements, the system is completely metered. It is found that the estimator converges for large noises. Of course, the error in the estimate is also very large. It should be noted

		CHANGE IN NOISE			CHANGE IN INITIAL GUESS			MINIMUM NUMBER OF BUSES	
Meter Type		E P Q	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q
Buses Metered		E: 1 2 3 4 5 P: 1 2 3 4 5 Q: 1 2 3 4 5	E: 1 2 3 4 5 P: 1 2 3 4 5 Q: 1 2 3 4 5	E: 1 2 3 4 5 P: 1 2 3 4 5 Q: 1 2 3 4 5	E: 1 3 5 P: 1 2 3 4 Q: 2 3 4 5	E: 1 3 5 P: 1 2 3 4 Q: 2 3 4 5	E: 1 3 5 P: 1 2 3 4 Q: 2 3 4 5	E: 1 3 5 P: 1 3 5 Q: 1 3 5	E: – P: 1 2 3 4 Q: 1 2 3 4 5
Measurement Error (%)		1 1 1	50 50 50	100 100 100	1 1 1	1 1 1	1 1 1	1 1 1	– 1 1
Error in Initial Guess (%)		5	0	0	1	50	100	1	1
SAMPLE ERROR IN ESTIMATE*	$\underline{\delta}_T - \hat{\underline{\delta}}$	.00001 .00004 .00006 .00006	.03198 .06914 .07692 .05488	.02255 .02115 .01557 .04456	.00053 .00097 .00111 .00137	.00053 .00097 .00111 .00137	.00053 .00097 .00111 .00132	No Convergence; Diverges	.00036 to .00082
	$\lvert E\rvert_T - \lvert\hat{E}\rvert$	.00029 .00005 .00091 .00082 .00096	.22337 .23076 .23371 .22525 .21743	.06411 .06192 .06007 .05993 .05687	.00273 .00315 .00317 .00312 .00271	.00273 .00315 .00317 .00313 .00271	.00273 .00315 .00317 .00312 .00271		.00392 to .00400

* in per-unit quantities

Table 6.2

Extreme Conditions for Estimator.

that the results listed in Table 6.2 are only from one sample, and not the average values. Hence, some inconsistencies will be found, such as the errors in the estimate not increasing in the same proportion as the increase in measurement noise. It is expected, though, that these inconsistencies would not be present if average values were listed.

In conclusion, the estimator can obtain an estimate of the state even for large measurement noises. The percent error of the estimate is large, but not significantly greater than and possibly less than the percent of the noise in the measurements.

LARGE INITIAL GUESS

When the initial guess of the state is in error by as much as 100%, the estimator still converges, and the number of iterations does not increase significantly. Moreover, the estimate is exactly the same for large errors in the initial guess as it is for small errors in the initial guess, provided that the same set of measurements is used in both cases. In conclusion, there does not seem to be any significant bound on the initial guess of the state which limits the ability of the estimator to converge.

MINIMUM NUMBER OF MEASUREMENTS

Table 6.2 clearly shows that the estimator is not limited by the minimum number of measurements, but by the minimum number of buses at which measurements are taken. This indicates again the importance of the meter placement problem. This problem is investigated later on.

6.3. Comparison of Σ with the Error Covariance Matrix.

The error covariance matrix is calculated by equation 6.1.

$$\underline{S} = \frac{1}{N} \sum_{i=1}^{N} (\underline{x}_T - \hat{\underline{x}})_i \, (\underline{x}_T - \hat{\underline{x}})_i^T$$

where N denotes the number of estimates obtained using different measurements.

Diagonal Elements	$\underline{S}^*$	$\underline{\Sigma}(\hat{\underline{x}})$	$\underline{\Sigma}(\underline{x}_T)$	$\underline{S}^*$	$\underline{\Sigma}(\hat{\underline{x}})$	$\underline{\Sigma}(\underline{x}_T)$
Measurement Noise (%)	1	1	1	10	10	10
$\left(\text{Per Unit Radians}\right)^2$ $\times 10^{-6}$	.4 1.0 1.0 2.0	.4 1.0 1.0 2.0	.47 1.3 1.5 2.1	42.0 102.0 117.0 187.0	43.0 123.0 139.0 197.0	47.0 128.0 146.0 218.0
$\left(\text{Per Unit Volts}\right)^2$ $\times 10^{-6}$	32.0 34.0 34.0 34.0 32.0	30.0 32.0 32.0 33.0 30.0	32.0 35.0 35.0 35.0 35.0	3120.0 3448.0 3442.0 3447.0 3184.0	3114.0 3385.0 3388.0 3397.0 3111.00	3316.0 3620.0 3623.0 3634.0 3319.0

(* for 30 trials)

Table 6.3

Σ versus the Error Covariance Matrix

The diagonal elements of the matrices $\sum(\hat{\underline{x}})$ and $\sum(\underline{x}_T)$ are compared with the diagonal elements of $\underline{S}$. The results clearly show that the matrix $\sum(\hat{\underline{x}})$ is indeed a good approximation to the error covariance matrix, even for measurement errors of 10%. Also, the bias errors of equation (3.4-5) are not significant. The results also show that the error covariance matrix $\underline{S}$ and the matrix $\sum$ increase in about the same proportion as the increase in measurement noise variance, as is expected.

Since $\sum(\hat{\underline{x}})$ is such a good approximation to the error covariance matrix, it is used in place of the error covariance matrix in the analysis of the estimator, provided the noise is small (within 10%) and the error in the estimate is small (within 10%). The tables for the remainder of the analyses list the square roots of the diagonal elements of $\sum(\hat{\underline{x}})$ as the average error in the estimate.

6.4. Coupling between P and δ and between Q and $|E|$.

It is mentioned in the introduction to this section that

$$\Delta \underline{P} \cong \underline{J}_1 \cdot \Delta \underline{\delta}$$

$$\Delta \underline{Q} \cong \underline{J}_4 \cdot \Delta |\underline{E}|$$

provided the X/R ratio is large. These approximations are studied in relation to the estimator by the following methods:

1. To study the coupling between watts and phase angles, the errors of all the real power measurements are increased while keeping the var measurement errors constant.
2. To study the coupling between vars and voltage magnitudes, the errors of the reactive power measurements are increased while keeping the watt measurement constant.

In both cases, no voltage measurements are taken. Table 6.4 lists the results of changing the real power measurement noises, and reactive power measurement noises.

It is expected that the approximations are not very good, because the network being used has a low X/R ratio of 3. The results in Table 6.4 confirm these suspicions. For example, the var measurements affect the phase angle estimates and the voltage magnitude estimates almost equally. Notice, however, that the error in the calculated reactive power is affected more than the error in the calculated real power. The real power measurements affects both the angle estimate and voltage estimate but unequally, and the errors do not increase in proportion to the increase in noise. However, the error of the calculated real power is affected more than the error of the calculated reactive power.

It is important to note that no reasonable estimate could be obtained twice with 10% noise in the watt measurements. This indicates that watt measurements can be critical in calculating an estimate. In conclusion then, the approximations that watt measurements do not affect the voltage estimate and that var measurements do not affect the phase angle estimate are very poor when the X/R ratio is 3. On the other hand, the watt measurements do not affect the calculated reactive powers, and likewise the var measurements do not affect the calculated real powers. The reason no voltage measurements are used in this study can be seen from the results of Table 6.4(b). It is found that now the watt measurements do affect the angle estimates more than the voltage magnitude estimates. This is because the voltage measurements are contributing to the voltage estimate, and hence keep the accuracy of the voltage estimate constant.

Meter Type		E P Q	E P Q	E P Q	E P Q	E P Q
Buses Metered		1 1 2 2 3 3 4 4 5 5	1 1 2 2 3 3 4 4 5 5	1 1 2 2 3 3 4 4 5 5	1 1 2 2 3 3 4 4 5 5	1 1 2 2 3 3 4 4 5 5
Measurement Noise (%)		1 1	1 10	1 20	5 1	10 1
Error in Estimate* $\sqrt{\sum \text{Diagonal}}$	$\Delta\delta_i$ i=1,4	.00034 .00060 .00063 .00078	.00318 .00548 .00581 .00675	.00633 .03550 .03770 .04380	.00079 .00134 .00133 .00216	.00316 .00505 .00530 .00736
	$\Delta\|E\|_i$ i=1,5	.00358 to .00365	.03090 to .03200	.05900 to .06090	.00973 to .01100	.00450 to .00470
Sample Error in Calculated Powers*	ΔP_i i=1,5	.00077 to .00611	.00068 to .00287	.00083 to .01204	.00200 to .02453	.00676 to .04296
	ΔQ_i i=1,5	.00004 to .00122	.00551 to .02215	.00164 to .01263	.00007 to .00168	.00010 to .00146

*in per-unit quantities

** could not obtain estimate twice out of 7 trials

Table 6.4(a)

Coupling between P and δ and between Q and $|E|$

	Meter Type	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q
	Buses Metered	1 1 2 2 3 3 3 4 4 5 5	1 1 2 2 3 3 3 4 4 5 5	1 1 2 2 3 3 3 4 4 5 5	1 1 2 2 3 3 3 4 4 5 5	1 1 2 2 3 3 3 4 4 5 5	1 1 2 2 3 3 3 4 4 5 5
	Measurement Noise (%)	1 1 1	1 10 1	1 20 1	1 1 10	1 1 20	1 1 80
Average Error in Estimate* $\sqrt{\Sigma'\text{ Diagonal}}$	$\delta_r - \delta_i$ i= 1,4	.00100 .00100 .00100 .00141	.00360 .00566 .00600 .00824	.00728 .01130 .01810 .01652	.00100 to .00141	.00100 to .00172	.00100 to .00245
	$\Delta\|E\|_i$ i = 1,5	.00574 .00600 .00600 .00600 .00574	.00566 to .00591	.00600 to .00762	.00600 to .00624	.00600 to .00640	.00624 to .00894
Sample Error in Calculated Powers*	ΔP_i i= 1,5	.00319 .00183 .00085 .00096 .00096	.03035 .00434 .01413 .03798 .02922	.01288 .05276 .00565 .06310 .00668	.00229 .00180 .00023 .00533 .00214	.00025 .00525 .00425 .00440 .00387	.00251 .00285 .00455 .00282 .00949
	ΔQ_i i=1, 5	.00696 .00069 .00047 .00107 .00007	.01248 .00153 .00078 .00170 .00015	.00995 .00076 .00005 .00142 .00145	.01294 .01323 .00047 .00846 .00335	.01720 .01175 .00382 .00986 .00541	.17746 .10033 .01120 .12270 .03596

*in per-unit quantities

Table 6.4 (b)

Effect of Voltage Measurements in Estimate

On the other hand, the var measurements affect both the voltage magnitude estimates and the phase angle estimates, but as the noise in the var measurements increases, the error in the estimate does not increase as fast. In this case, the combination of voltage measurements and watt measurements are weighted more heavily than the var measurements as the noise in the var measurements increases. Thus, the estimate is being calculated more from the watt and voltage measurements, than from var measurements.

These results, then, indicate that the voltage magnitude measurements contribute significantly to the estimate of the state, especially when some of the power measurements has a high noise level.

6.5. Importance of Meter Type

The relative importance of each of the three types of meters is studied by eliminating one type of meter at a time from the observation vector. A reference is established by completely metering the system. The results in Table 6.5 show that the wattmeters are the most important type of meter. The varmeters are next in importance, while the voltmeters have the least importance. In fact, the estimate obtained with no voltmeters is almost as accurate as that with complete metering.

The varmeters are not as important as the wattmeters, because the voltmeters also contribute in the estimate of the voltage magnitudes. This contribution, however, does not help enough, because the calculated reactive powers have errors greater than 100%.

	Meter Type	E	P	Q	E	P	Q	E	P	Q	E	P	Q	E	P	Q	E	P	Q	E	P	Q
	Buses Metered	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5		1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5		1 2 3 4 5		1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
	Measurement Noise %	1	1	1		1	1	1	1		1		1	1	10	1	1	1	10	10	1	1
Error in Estimate* $\sqrt{\sum \text{Diagonal}}$	$\Delta\delta_i$ i= 1, 4	0.00028 to 0.00071			.00034 to .000775			.0049 to .0052			No Convergence			.00100 to .00280			.00100 ↓			.00031 to .00055		
	$\Delta\lvert E\rvert_i$ 1 = 1, 5	0.00283 ↓			.00355 ↓			.010 ↓						.0041 ↓			.00447 ↓			.00360 ↓		
Sample Error in Calculated Powers*	ΔP_i i = 1, 5	0.00199 to 0.00463			.00164 to .00693			.00193 to .01030						.00184 to .05605			.00074 to .01080			.00110 to .01138		
	ΔQ_i i = 1, 5	0.00013 to 0.00091			.00026 to .00116			.14556 to .48030						.00003 to .00301			.00027 to .01085			.00020 to .00097		

* in per unit quantities

TABLE 6.5

IMPORTANCE OF METER TYPE

These same studies are carried out again, but now by increasing the meter noise instead of by eliminating the meters. The results in Table 6.5 indicate that changes in both watt measurement errors and var measurement errors affect the accuracy of the estimate much more than do the changes in the voltage measurement errors, In fact, the error of the estimate with no voltage measurements is almost exactly the same as with 10% voltage measurement noise. This indicates that the voltage measurements are weighted very little with respect to the watt measurements, and the var measurements.

6.6. Increasing the Number of Var Meters.

Table 6.6 shows the results of increasing the number of varmeters from zero to three. It should be noted that the errors in the powers calculated from the estimate are only the results of one sample, whereas the errors in the estimate are average values. Some inconsistencies are to be expected, in the listings of the calculated power errors. Also it should be remembered that the example network used in these studies has a low (X/R) ratio which causes cross coupling between P and $|E|$ and between Q and δ. This effect is discussed in section 6.4.

The results clearly indicate that the calculated real powers do not depend on the var measurements, which is expected. However, the phase angle estimate does improve when more varmeters are used. This is caused by the low (X/R) ratio.

On the other hand, the calculated reactive powers and the voltage magnitude estimates are strongly affected by the var measurements, especially at the buses where the varmeters are placed. For instance, when a varmeter is placed at bus 5 both the voltage magnitude error and the calculated reactive power error at bus 5 decrease significantly. It is also found that there is a slight

Meter Type	|E| P Q	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q
Buses Metered	1 1 2 2 3 3 4 4 5 5	1 1 2 2 3 3 4 4 4 5 5	1 1 2 2 3 3 4 4 5 5 5	1 1 1 2 2 3 3 4 4 5 5 5	1 1 2 2 3 3 3 4 4 5 5 5	1 1 1 2 2 3 3 3 4 4 5 5 5	1 1 1 2 2 2 3 3 3 4 4 4 5 5 5
Measurement Noise (%)	1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
Error in Estimate* $\sqrt{\sum \text{Diagonal}}$: $\Delta \delta_i$, i=1,4	.00495 .00504 .00507 .00498	.00463 .00497 .00512 .00415	.00094 .00358 .00453 .00417	.00084 .00186 .00360 .00295	.00071 .00360 .00300 .00435	.00062 .00152 .00115 .00268	.00028 .00044 .00044 .00071
$\Delta \|E\|_i$, i=1, 5	.01040 .01015 .01010 .01020 .01060	.00866 .01020 .00985 .00627 .01060	.00803 .00980 .01010 .01010 .00655	.00465 .00894 .01000 .00952 .00490	.00811 .00763 .00588 .01010 .00640	.00434 .00582 .00486 .00950 .00422	.000283 ↓
Sample Error In Calculated Powers*: ΔP_i, i=1, 5	.00150 .00385 .00405 .00030 .00089	.00116 .00673 .00336 .00404 .00686	.00193 .00186 .00645 .00419 .01113	.00114 .00673 .00342 .00400 .00691	.00333 .00407 .00107 .00594 .00720	.00316 .00114 .00173 .01316 .01182	.00149 to .00463
ΔQ_i, i= 1, 5	.22118 .43639 .38109 .14282 .02017	.42050 .16792 .00856 .00061 .25552	.28804 .41007 .31472 .18506 .00123	.00120 .15733 .10767 .05630 .00049	.22908 .05768 .00024 .16564 .00038	.00437 .04889 .00023 .03340 .00065	.00013 to .00091

* in per unit quantities

(reference)

Table 6.6.

Increasing Number of Q Measurements

From 0 to 3.

decrease in the voltage magnitude error at those buses which are close to the metered buses. For instance, buses 1 and 5 are connected by a short transmission line, and when a varmeter is placed at bus 5, the voltage magnitude error at bus 5 decreases and is accompanied by a slight decrease in the voltage error at bus 1. This "spreading effect", in which the measurements at one bus affect the estimate of the state at other buses, is very important, because it can mean that the system is capable of being "probed" by the meters.

The errors of the estimate are less than 1% when three varmeters are used. Even the errors of the calculated reactive powers at the buses which have no varmeters decrease significantly. This is due to the contribution of the "spreading effect" of the measurements on the accuracy of the estimate, and also the contribution of the voltage magnitudes in the voltage estimate. It is expected that even further improvements in the estimate can be obtained if psuedo var measurments are used at the buses which have no varmeters. (See the results of section 6.8).

6.7. Increasing the Number of Wattmeters

In conjunction with increasing the number of varmeters, the number of wattmeters is also increased from zero to three. The results are shown in Table 6.7.

An estimate cannot be obtained when there are less than two wattmeters. This confirms the results of section 6.5, that wattmeters are very necessary in obtaining an estimate of the state.

An estimate is obtained when two or more wattmeters are used. There are definite improvements in the phase angle estimate and in the calculated real powers, especially at the buses where wattmeters are placed. This localized

Meter Type		E P Q	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q
Buses Metered		E: 1 2 3 4 5 P: Q: 1 2 3 4 5	E: 1 2 3 4 5 P: 4 Q: 1 2 3 4 5	E: 1 2 3 4 5 P: 5 Q: 1 2 3 4 5	E: 1 2 3 4 5 P: 1 5 Q: 1 2 3 4 5	E: 1 2 3 4 5 P: 3 5 Q: 1 2 3 4 5	E: 2 3 4 5 P: 1 3 5 Q: 1 2 3 4 5	E: 1 2 3 4 5 P: 1 2 3 4 5 Q: 1 2 3 4 5
Measurement Noise (%)		1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
Error in Estimate* $\sqrt{\sum \text{Diagonal}}$	$\Delta\delta_i$ i=1,4	no convergence	no convergence	no convergence	.00271 .01121 .00775 .00728	.0052 .0202 .0137 .0139	.00045 .00425 .00281 .00574	.00028 .00044 .00044 .00071
	$\Delta\|E\|_i$ i=1,5	Estimate Blows Up	Estimate Blows Up, Cannot	Estimate Cannot be Obtained	.00481 .00678 .00673 .00943 .00476	.00583 .00714 .00531 .00812 .00480	.00446 .00583 .00539 .00330 .00469	.00283 ↓
Sample Error In Calculated Powers*	ΔP_i i=1,5		Be Obtained		.00056 .69704 .63932 .07600 .01760	.16104 .25099 .00160 .00930 .00013	.00000 .08377 .00150 .08373 .00065	.00149 to .00463
	ΔQ_i i=1,5				.00091 .00143 .00099 .00260 .00008	.00032 .00130 .00000 .00080 .00060	.00225 .00091 .00051 .00253 .00029	.00013 to .00091

* in per-unit quantities

(reference)

Table 6.7

Changing Number of P Measurements
From Zero to Three

improvement of the estimate by the measurements is also noticed in the results of section 6.6. When three wattmeters are used, it is found that the errors of the estimate of the state and of the calculated real powers decreased at the buses which do not have wattmeters. This is due to the "spreading effect" discussed in the previous section.

The watt measurments do not affect the calculated reactive powers, as is to be expected. There is, however, some improvement of the voltage magnitude estimates. This is caused by the coupling between real power measurements and the voltage magnitudes due to the low (X/R) ratio.

6.8. Minimum Number of Buses

In this section, the minimum number of buses that can be measured using all the three meter types is studied. Since there are nine unknowns, nine observations must be taken, and so three buses must be metered. Three buses are choosen (1, 3, 5), and then four buses are metered which include the three choosen buses and an extra bus (bus 2). The noise of the measurments at bus 2 is increased to observe the effect of gradually eliminating the measurements at that bus by weighting them out of the estimate.

The results shown in Table 6.8 clearly indicate that as the error in the measurments at bus 2 increase, the errors in the estimate of the state increase both at bus 2 and at bus 4 which has no meters. However, the estimates of the state at the other buses do not change. It is also clear that the errors of the real and reactive power at buses 2 and 4 increase almost as fast as the noise increases in the meters at bus 2, while the errors in the calculated powers at the other buses are hardly affected.

Meter Type		E P Q	E P Q	E P Q	E P Q	E P Q	E P Q
Buses Metered		1 1 1 3 3 3 5 5 5	1 1 1 2 2 2 3 3 3 5 5 5	1 1 1 2 2 2 3 3 3 5 5 5	1 1 1 2 2 2 3 3 3 5 5 5	1 1 1 2 2 2 3 3 3 5 5 5	1 1 1 2 2 2 3 3 3 5 5 5
Measurement Noise: Bus 2 (%)		1 1 1	1 1 1	10 10 10	50 50 50	100 100 100	1 1 1
Error In Estimate* $\sqrt{\Sigma \text{ Diagonal}}$	$\Delta\delta_i$ I=1,4	no convergence	.00072 .00103 .00113 .00197	.00084 .00192 .00145 .00400	.00214 .00805 .00440 .01730	.00430 .01814 .00980 .03620	.00028 .00044 .00044 .00071
	$\Delta\|E\|_i$ i=1,5	cannot obtain estimate	.00470 to .00500	.00534 .00550 .00548 .00548 .00546	.00511 .00628 .00564 .03320 .00544	.00667 .01083 .00816 .02010 .00628	.00221 ↓
Sample Error In Calculated Powers*	ΔP_i i=1,5		.00007 to .00204	.00202 .01270 .00400 .01867 .00888	.00021 .08598 .00433 .10129 .01185	.00227 .54090 .00192 .53824 .00900	.0019 to .0046
	ΔQ_i i=1,5		.00057 to .00387	.00094 .02361 .00014 .01917 .00051	.00180 .13212 .00043 .13005 .00062	.00270 .15302 .00001 .17490 .00072	.00013 to .00091

* in per unit quantities

(reference)

Table 6.8

Minimum Number of Buses
(1% error for all measurements except bus 2)

The percent error of the estimate for 50% and 100% meter noise at bus 2 do not match the error in the calculated power, because the approximation of the error covariance matrix by $\sum(\hat{\underline{x}})$ is not as good.

The estimate does not converge when only three buses are metered. Obviously, three buses do not contain enough information to estimate the state, even though there are enough measurements. The reason for this is that there is some redundancy in measuring voltage and reactive power at the same bus since they are coupled. This redundancy is investigated in section 6.9.

In conclusion, the estimator cannot obtain an estimate by metering a minimum number of buses (at least for this example network). When the noise of the measurement at one bus is increased, the estimate at that bus and any unmetered buses becomes less accurate. This "localized effect" of the measurements is also found in section 6.6.

6.9 Voltage Measurements in Relation to Var Measurements

It is found that the reactive power measurements affect the voltage magnitude estimate. It is also found that the var measurements are more important than the voltage measurements. However, the voltage measurements do contribute to the estimate of the voltage estimate, especially when var measurements have high noise levels or are not present.

The ability of the voltage measurements to aid the var measurements in estimating the state is studied, and the results are shown in Table 6.9. The first part of the investigation is conducted by placing the voltmeters at the buses where there are no varmeters. Next the voltmeters are placed where there are varmeters. The results show that the voltage magnitude estimate does improve at the buses where voltmeters are placed, but not significantly more than when voltmeters are placed only where the varmeters are located. However, the calculated reactive powers do improve significantly when voltmeters are placed where there is no varmeter .

This study is continued by using psuedo measurements at two of the buses (buses 2 and 3). (Psuedo measurements are guesses of the power calculated from past data. A psuedo measurement has a much higher error than a direct measurement, e. g. 50%). The results indicate that considerable improvement is obtained when voltmeters are placed at the bus where psuedo var measurements are used. In addition, the calculated reactive power also becomes more accurate at these buses.

It is interesting to note that the estimate using psuedo-var measurements is better than the estimate obtained with no var measurements at the same buses. This indicates that the estimator does indeed use the information of the psuedo measurements to improve the estimate.

Meter Type	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q	E P Q
Buses Metered (E)	2	3	2 3	1 4 5	2 3	2 3	2 3		1 2 3 4 5
Buses Metered (P)	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
Buses Metered (Q)	1 4 5	1 4 5	1 4 5	1 4 5	1 2* 3* 4 5	1 2 3 4 5	1 2* 3* 4 5	1 2* 3* 4 5	1 2 3 4 5
Measurement Noise (%)	1 1 1	1 1 1	1 1 1	1 1 1	1 1 20*/1	1 1 40*/1	1 1 60*/1	1 20*/1	1 1 1
Error In Estimate[1] $\sqrt{\Sigma \text{Diagonal}}$ $\Delta\phi_i$ i=1,4	.00115 .00954 .00540 .00245	.00158 .00210 .00487 .00383	.00104 .00192 .00362 .00221	.00077 .00268 .00324 .00195	.00073 .00136 .00141 .00163	.00073 .00138 .00143 .00165	.00075 .00141 .00147 .00168	.00291 .00548 .00572 .00644	.00028 .00044 .00044 .00071
$\Delta\lvert E\rvert_i$ i=1,5	.00752 .01030 .04580 .03330 .01970	.01410 .02080 .01030 .01275 .01525	.00728 .01010 .01010 .00784 .00740	.00602 .01064 .01020 .00657 .00640	.00700 to .00728	.00710 to .00740	.00726 to .00757	.03010 .02980 .03010 .03100 .02980	.00283
Sample Error In Calculated Power[1] ΔP_i i=1,5	.00110 .01533 .00434 .00130 .00680	.00163 .00004 .00274 .00340 .02635	.00107 .00036 .00019 .00175 .00035	.00193 .00341 .00160 .00252 .00465	.00114 .00095 .00275 .00520 .00833	.00329 .00014 .00374 .00169 .00110	.00192 .00516 .00716 .00435 .01535	.00386 .00156 .00345 .00456 .00729	.00149 to .00463
ΔQ_i i=1,5	.00240 .88490 .90220 .00003 .00010	.00073 1.53320 1.48040 .00065 .00126	.00675 .24690 .26260 .00060 .00085	.00210 1.35756 1.31900 .00157 .00070	.00103 .00494 .00235 .00016 .00037	.00103 .00008 .00032 .00002 .00029	.00050 .00103 .00165 .00076 .00036	.00113 .00654 .00183 .00026 .00086	.00013 to .00091

[1] in per unit quanties

(reference)

Table 6.9

|E| Metering in Relation to Q Metering

(*indicates those buses where psuedo measurements are used and the noise of the psuedo measurements)

6.10. Summary of Conclusions

The following list is a summary of the major conclusions based in the analysis of the estimator.

1. For a well metered system, and for low noise levels, the estimator program converges to a good estimate of the state. The percent error of the estimate can be less than the percent of the noise in the measurements.

2. The estimator can still obtain good estimates for large errors in the initial value of the state, and when the minimum number of measurements is used. The estimator will converge even for large measurement noises provided the system is well metered; the error in the estimate is large, however.

3. The error covariance matrix can be approximated by the matrix $\sum(\hat{\underline{x}})$ for small measurements noise levels and for small errors in the state.

4. The approximation that watt measurements do not affect the voltage magnitude estimate does not hold for an X/R ratio of 3. Likewise, the approximation that var measurements do not affect the phase angle estimate does not hold for an X/R ratio of 3.

5. The three meter types can be arranged according to their relative importance in contributing to an estimate. The wattmeters are most important; the varmeters are next in importance; the voltmeters have the least importance.

6. There are circumstances, however, where voltmeters improve the accuracy of the estimate significantly. Voltmeters usually "help out" the varmeters in obtaining an estimate of the voltage magnitudes, especially when some of the var measurements are more inaccurate than the voltage measurements or are not present.
7. The measurements at a bus are most important for calculating the estimate of the state at that bus; measurements have localized affect. There does seem to be some "spreading effect" of the measurements, especially for buses which are close together.
8. Psuedo measurements are better than no measurements in calculating the estimate of the state.

Section 7. Proposals for Further Study

The performance of the state estimator needs to be analyzed more thoroughly; the studies undertaken in this thesis are just a beginning. Some important topics which need to be investigated are now discussed.

The three major topics of the affects of meter placement, meter type, and meter noise on the accuracy of the estimate still need to be studied. Some ideas are listed below:

1. The estimator program can be altered to include the powers flowing in the lines as measurements. These additional measurements are expected to increase the accuracy of the estimate. Moreover, the problem of "probing" the system can be studied in detail. By "probing" it is meant that measurements of powers flowing in the lines toward an unmetered bus are used to estimate the state at that bus. Power companies are very interested in this problem, for it would mean that fewer meters are necessary.

   Power companies are also interested in monitoring the power flows in the lines to determine if a line is overloaded, etc. Calculating the power flows in the lines from the estimates gives an indication of the magnitude of the error in the line powers measured. Hence, the reliability of these measurements is checked.

2. The reactance to resistance ratio (X/R) of the model can be increased to ten or more. It is expected that at this ratio the coupling between the var measurements and the voltage magnitude estimate, and also between the watt measurements and the phase angle estimates should be very strong. This can be easily checked from the computer simulation.

If the results do show that the coupling is strong, then it is suggested that the Jacobian matrix $\underline{H}_x$ in the estimator be approximated by

$$\underline{H}_x = \begin{bmatrix} \underline{0} & \frac{\partial E_i}{\partial E_j} \\ \underline{J}_1' & \underline{0} \\ \underline{0} & \underline{J}_2' \end{bmatrix}$$

The feasibility of this approximation can be determined from the simulation program. If it does work, some computer time will be saved.

3. The larger (X/R) ratio also stresses the interrelation between the var measurements and the voltage measurements in obtaining an estimate of the state. Additional studies will provide some insight into this relation.

The estimator program itself can be studied. Three items of interest are the following:

1. Instead of recomputing the gain matrix ($\Sigma' \underline{H}_x \underline{R}^{-1}$) in every iteration, it is suggested that a pre-calculated constant gain matrix be used. If this is feasable, then a large amount of computer time will be saved.

2. The conditions which cause the estimator to converge in a larger number of iterations than usual, or even to diverge are not thoroughly understood. It seems that the magnitude of the errors in the measurements have the most important affect. Both theoretical and numerical studies of the estimator equation can give some insight into this question.

3. The estimator program in these studies uses a "snapshot" observation of the system. However, for larger networks, the computer system may not be able to handle all the measurements simultaneously. An alternate method is to send the measurements to the computer sequentially in time. This sequential observation mode can be implemented on the simulation program. The program should include a method of updating the measurements and provisions for altering the scan rate.

There are several topics related to the estimator program which can be studied. Two of them are:

1. Investigating the role of the estimator in the detection of network changes. For instance, the detection of a transmission line loss, or generator failure can employ the estimator program. Another example is the detection of a meter failure. In fact, the application of detection theory or hypothesis testing to power systems is an area of study in itself.

2. Investigating another means of estimating the state. In particular, the use of unknown-but-bounded estimation theory discussed in reference (4) may prove feasible.

3. The tie lines connecting one company to the adjoining companies should be taken into account, because changes in one own's system cause changes in the tie line power flows. A method of predicting the tie line flow changes has been developed (6). The estimator program would be more complete if this prediction program were included.

Concluding Remarks

The purpose of the thesis is to construct and analyze a static state estimation program for a bulk transmission system. The computer simulation program is contructed and does work quite well. The estimation program has been checked out in some detail, and the results show that the estimator does calculate an accurate estimate of the state given enough measurements with low noise levels. Hence, the first objective has been completed.

The simulation program is then used to analyze the performance of the estimator under various conditions. The essential results are listed in Tables 6.1 through 6.9 and an analysis of these results is presented in Section 6. The analysis has been facilitated by using a scaled-down version of a bulk transmission network model, and by approximating the error covariance matrix by a matrix which is calculated automatically in the estimator program.

The results show that the coupling between watt measurements and the phase angle estimate, and also between the var measurements and the voltage magnitude estimate is not very strong at an X/R ratio of three to one. It is also found that, relatively speaking, the watt measurements are most important, var measurements are next, and voltage measurements are the least. There is some relationship between the var measurements and the voltage measurements, but the results are not conclusive. Furthermore, it is found that the measurement at a bus have a localized affect on the estimate; that is, they are most important for calculating the state at that bus. The problem of "probing" the system was not investigated per se, but the results do seem to indicate that the system can be "probed". Also the affect of psuedo measurements is investigated. The results show that psuedo measurements do help obtain a better estimate. However, more studies need to be made in this area, too. A complete summary of the results is found in Section 6.10.

The investigations undertaken are just a beginning; much more work needs to be done. Thus, the second objective of this thesis has not really been completed. However, the results obtained are perhaps most useful by indicating what areas should be investigated, and how these studies should be undertaken.

Appendix A: Example Model

The example model is taken from Stagg and El Abiad (1). The model represents a 138kv bulk transmission system. It has 5 buses, 7 lines, 2 generators and 4 loads. See Figure A-1 for the network diagram.

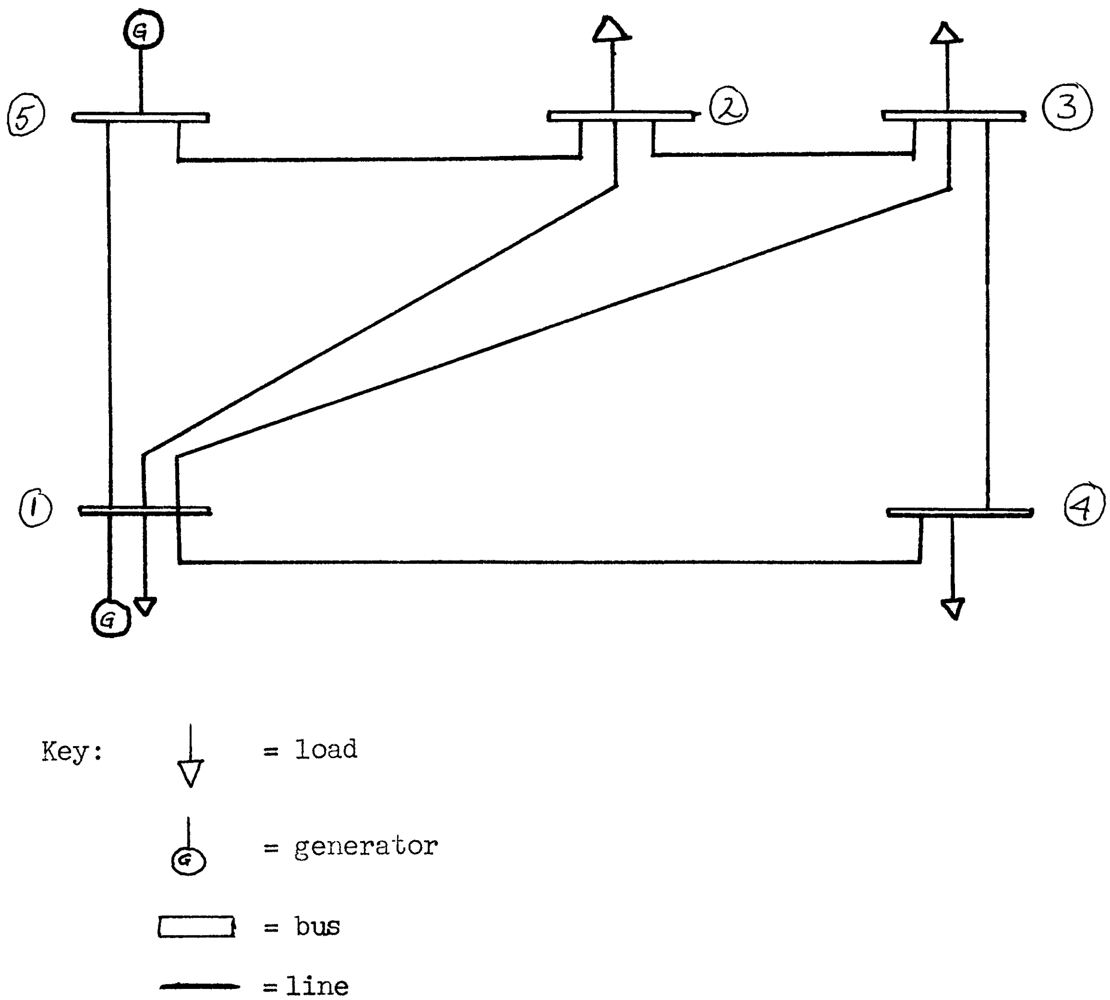

Fig. A-1

Example Model

Bus Code	Line Admittance*	Bus	Shunt Admittance* to ground
1-2	1.66667-j5.00000	1	0.0 + j0.08500
1-3	1.66667-j5.00000	2	0.0 + j0.05500
1-4	2.50000-j7.50000	3	0.0 + j0.05500
1-5	5.00000-j15.00000	4	0.0 + j0.04000
2-3	10.00000-j30.00000	5	0.0 + j0.05500
3-4	1.25000-j3.75000		
2-5	1.25000-j3.75000		

(* 100,000 kva base)

TABLE A-1.

Line Admittances and Shunt Admittances.

$$
\begin{bmatrix}
34.17738\,e^{j1.24826} & 5.27046\,e^{j4.39064} & 5.27046\,e^{j4.39064} & 7.90569\,e^{j4.39064} & 15.81139\,e^{j4.39064} \\
5.27046\,e^{j4.39064} & 40.79391\,e^{j1.24862} & 31.62278\,e^{j4.39064} & 0.0 & 3.95385\,e^{j4.39064} \\
5.27046\,e^{j4.39064} & 31.62278\,e^{j4.39064} & 40.79391\,e^{j1.24862} & 3.95285\,e^{j4.39064} & 0.0 \\
7.90569\,e^{j4.39064} & 0.0 & 3.95285\,e^{j4.39064} & 11.82060\,e^{j1.24798} & 0.0 \\
15.81139\,e^{j4.39064} & 3.95285\,e^{j4.39064} & 0.0 & 0.0 & 19.71207\,e^{j1.24816}
\end{bmatrix}
$$

Table A-2

Bus Admittance Matrix in Polar Form

Appendix B: Example of Load Flow Calculations

The load flow calculations use the example model of Appendix A. Bus 5 is choosen as the slack bus. The scheduled powers are listed in Table B-1. The initial values of voltage magnitude and angle are listed in Table B-2. The calculated voltage magnitudes and angles, as well as the powers at all the buses are listed in Table B-3.

	Generation		Load	
Bus	Megawatts	Megavars	Megawatts	Megavars
1	0	0	0	0
2	40	30	20	10
3	0	0	45	15
4	0	0	40	5
5	0	0	60	10

TABLE B -1.

Scheduled Powers

Bus	Voltage Magnitude $\lvert E \rvert$	Phase Angle $\lvert \delta \rvert$
1	1.0	0.0
2	1.0	0.0
3	1.0	0.0
4	1.0	0.0
5	1.06	0.0

TABLE B -2

Initial Values of the State
(on a 100 KVA Base) (138KV)

Bus	Voltage $\lvert E \rvert_T$	Voltage δ_T	Power P_T	Power Q_T
1	1.04744	-.04898	.20000	.20000
2	1.02418	-.08722	-.45000	-.15000
3	1.02357	-.09302	-.40000	-.05000
4	1.01794	-.10734	-.60000	-.10000
5	1.06000	0.00000	-1.29604	-.07426

Table B—3

True Values of the State and Powers
(On a 100 KVA Base) (138 KV)

Appendix C: Linear Fisher Estimation Theory

Consider the following equation:

$$\underline{z} = \underline{H}\ \underline{x} + \underline{v} \tag{C-1}$$

where $\underline{x}$ is a n vector

$\underline{z}$ is a m vector

$\underline{v}$ is a m vector

$\underline{H}$ is a m x n matrix

$\underline{H}$ is known; the state $\underline{x}$ is unknown; $\underline{v}$ is the "noise"; and $\underline{z}$ is observed. This linear equation is considered to be time invarient.

Fisher estimation theory obtains the best estimate of the state $\hat{\underline{x}}$ from the observations $\underline{z}$. The noise $\underline{v}$ is modeled as a random vector. Assume

$$\begin{aligned} E\ (\underline{v}) &= \underline{0} \\ E\ (\underline{v}\ .\ \underline{v}^T) &= \underline{R} \end{aligned} \tag{C-2}$$

and that $m \geqslant n$. If the value of the vector $\underline{x}$ is known, and the probability density of $\underline{v}$ is known, then the probability density of $\underline{z}$ can be calculated. Let $p\ (\underline{z} : \underline{x})$ denote the probability density of $\underline{z}$ given $\underline{x}$. This probability density is called the likelihood function. The "best" estimate of the state is defined as the value of $\underline{x}$ which maximizes $p\ (\underline{z} : \underline{x})$ for some given observation $\underline{z}_0$.

The usual case is that $\underline{v}$ has a Gaussian probability density. That is

$$p\ (\underline{v}) = \frac{1}{\sqrt{2\pi |\underline{R}|}}\ e^{-\frac{1}{2}\underline{v}^T \cdot \underline{R}^{-1} \cdot \underline{v}} \tag{C-3}$$

Hence, $p\,(\underline{z}:\underline{x}) = \dfrac{1}{\sqrt{2\pi\,|\underline{R}|}}\; e^{-\frac{1}{2} Q\,(\underline{x})}$ (C-4)

where $\underline{Q}\,(\underline{x}) = \left[\underline{z} - \underline{H}\cdot\underline{x}\right]^T \cdot \underline{R}^{-1} \cdot \left[\underline{z} - \underline{H}\cdot\underline{x}\right]$ (C-5)

Since $Q\,(\underline{x})$ is non-negative, then the value of $\underline{x}$ which maximizes $p\,(\underline{z} : \underline{x})$ minimizes $Q\,(\underline{x})$.

There are several ways to minimize $Q(\underline{x})$ with respect to $\underline{x}$. One method is to rewrite (C-5) after some manipulation as

$$Q\,(\underline{x}) = \underline{z}^T \cdot \left[\underline{R}^{-1} - \underline{R}^{-1}\,\underline{H}\,\underline{\Sigma}\,\underline{H}^T\,\underline{R}^{-1}\right] \cdot \underline{z}$$

$$+ \left[\underline{x} - \underline{\Sigma}\,\underline{H}^T\,\underline{R}^{-1}\,\underline{z}\right]^T\,\underline{\Sigma}^{-1}\,(\underline{x} - \underline{\Sigma}\,\underline{H}^T\,\underline{R}^{-1}\,) \qquad \text{(C-6)}$$

where $\underline{\Sigma} = \left[\underline{H}^T\,\underline{R}^{-1}\,\underline{H}\right]^{-1}$ (C-7)

The matrix $\underline{\Sigma}$ is assumed to exist. Since the second term of (C-6) is non-negative the value of $\underline{x}$ which minimize $Q\,(\underline{x})$ is

$$\underline{\hat{x}} = \left[\underline{H}^T \cdot \underline{R}^{-1} \cdot \underline{H}\right]^{-1} \cdot \underline{H}^T \cdot \underline{R} \cdot \underline{z} \qquad \text{(C-8)}$$

Thus (C-8) gives the maximum likelihood estimate if $\underline{v}$ is Gaussian. Another way to obtain (C-8) is to take the partial derivative of $Q\,(\underline{x})$ with respect to $\underline{x}$, set the result equal to zero, and solve for $\underline{\hat{x}}$.

Some properties of (C-8) are now derived:

$$E\,[\underline{\hat{x}}] = \left[\underline{H}^T\,\underline{R}^{-1}\,\underline{H}\right]^{-1} \cdot \underline{H}^T\,\underline{R}^{-1} \cdot E\,[\underline{z}]. \qquad \text{(C-9)}$$

but $E\,(\underline{z}) = \underline{H}\cdot\underline{x}$, (C-10)

since $E\,(\underline{v}) = \underline{0}$

so $E\,(\underline{\hat{x}}) = \left[\underline{H}^T\,\underline{R}^{-1}\,\underline{H}\right]^{-1} \cdot \underline{H}^T \cdot \underline{R}^{-1} \cdot \underline{H}\cdot\underline{x} = \underline{x}$ (C-11)

In other words, the estimate is unbiased. ALso,

$$(\underline{x} - \underline{\hat{x}}) = \underline{x} - \underline{\Sigma} \cdot \underline{H}^T \cdot \underline{R}^{-1} \cdot \underline{z} \qquad \text{(C-12)}$$

$$= \underline{x} - \underline{\Sigma}\, \underline{H}^T \underline{R}^{-1} (\underline{H}\ \underline{x} + \underline{v}) = - \underline{\Sigma}\ \underline{H}^T\ \underline{R}^{-1}\ \underline{v}$$

hence

$$E \left[(\underline{x} - \hat{\underline{x}}) (\underline{x} - \hat{\underline{x}})^T \right] = \underline{\Sigma}\, \underline{H}^T\ \underline{R}^{-1}\ (\ E\ (\underline{v}\underline{v}^T)\ )\ \underline{R}^{-1} \underline{H}^T\ \underline{\Sigma} \qquad \text{(C-13)}$$

but $E\ (\underline{v}\ \underline{v}^T) = \underline{R}$

so $E\ (\ (\underline{x} - \hat{\underline{x}})\ (\underline{x} - \hat{\underline{x}})^T\ ) = \underline{H}^T\ \underline{R}^{-1}\ \underline{R}\ \underline{R}\ \underline{R}^{-1}\ \underline{H}^T = \Sigma$ (C-14)

In other words, the matrix $\underline{\Sigma}$ equals the error covariance matrix.

Appendix D. Computer Programs Listings

In this Appendix, the listings of all the programs are presented. The Load Flow program, the estimator program and some of the more complicated subroutines have short descriptions which explain the notation. A flow chart for the load flow is given in Figure D-1 and for the estimator program in Figure D-3.

LOAD FLOW

The computer program for the load flow calculations is set up to solve the following 2(n-1) equations.

$$\begin{bmatrix} \Delta \underline{P}_i \\ \Delta \underline{Q}_i \end{bmatrix} = \begin{bmatrix} \underline{J}_1 & \underline{J}_2 \\ \underline{J}_3 & \underline{J}_4 \end{bmatrix} \cdot \begin{bmatrix} \Delta \underline{\delta}_i \\ \Delta |\underline{E}|_i \end{bmatrix}$$

$i=1, n-1,$

where bus n is the slack bus, and where $\underline{J}_1$, $\underline{J}_2$, $\underline{J}_3$ and $\underline{J}_4$ are the corresponding elements of the Jacobian. (See equations (2.3-8) through (2.3-11)). The flow chart on Fig. (D-1) describes the operation of the program. A program listing is found on Fig (D-2).

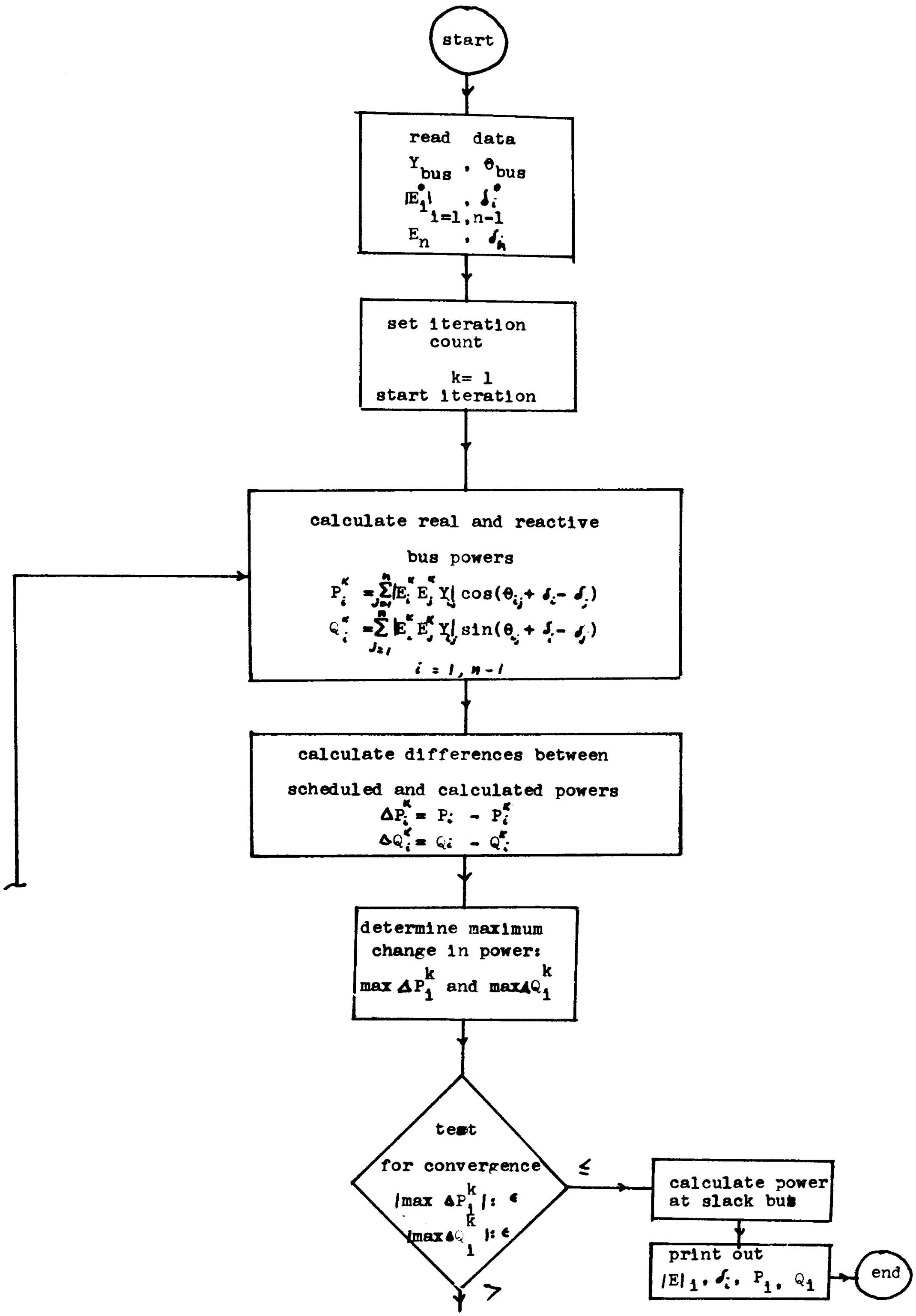

Fig. D-1

Flow Chart for Load Flow Calculations

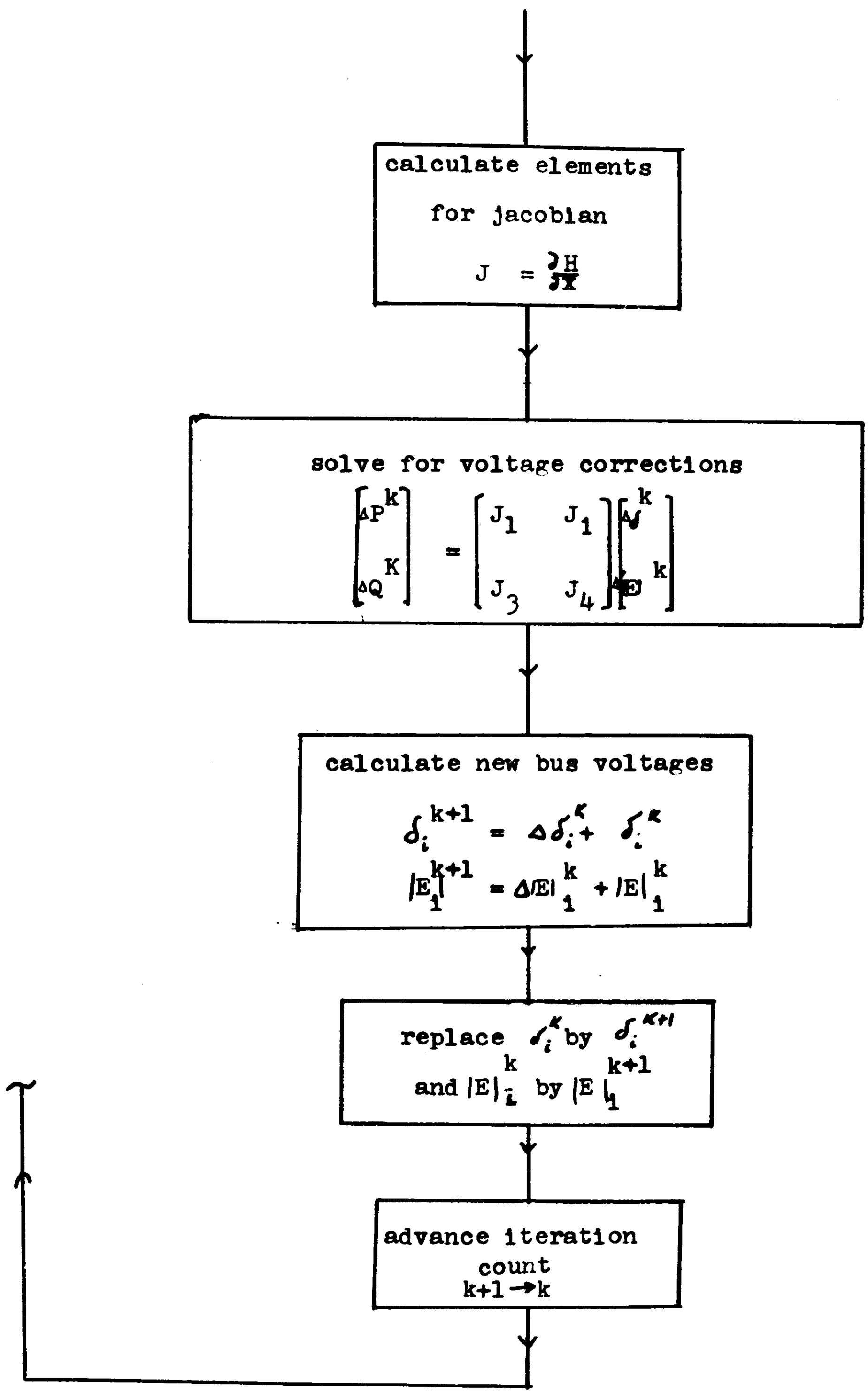

Fig. D-1
(continued)

```
C     THIS IS THE MAIN PROGRAM FOR LOAD FLOW CALCULATIONS
C
      REAL J
      DIMENSION Y(5,5), T(5,5),ET(5),DT(5),P(5),Q(5),DP(5)
      DIMENSION  DQ(5),PSKED(5),QSKED(5),J(8,8),DPWR(8)
      READ (5,4) N,L,EPS
      READ (5,1) ((Y(I,K),T(I,K), K =1,N), I =1,N)
      READ (5,1) (PSKED(I), QSKED(I), I = 1,N)
      READ (5,1) (ET(I),DT(I), I =1,N)
      M = N-1
      M2 = M*2
      NN2 = N*2
1     FORMAT(1X,4F11.5)
4     FORMAT(2I2,F10.7)
      DO 10 I  = 1,L
      CALL POWER(P,Q,ET,DT,Y,T,N)
      CALL VSUM(PSKED,P,DP,-1.0,M)
      CALL VSUM(QSKED,Q,DQ, -1.0,M)
      REAL MAX
      DPMAX = MAX(DP,M)
      DQMAX = MAX(DQ,M)
      IF(DPMAX .LE. EPS .AND. DQMAX .LE. EPS) GO TO 11
      CALL JACOBN(J,ET,DT,Y,T,N,NN2)
      DO 12 K = 1,M
      DPWR(K) = DP(K)
      KK = K + N - 1
      DPWR(KK)  = DQ(K)
12    CONTINUE
      CALL SIMQ (J,DPWR,M2,KS,M2)
      IF (KS .GE. 1) GO TO 15
      DO 13 K = 1,M
      DT(K) = DT(K) + DPWR(K)
      KK = K + N - 1
      ET(K) = ET(K) + DPWR(KK)
13    CONTINUE
10    CONTINUE
      GO TO 20
11    WRITE (6,9) ,I
9     FORMAT(13HCONVERGES IN ,I2,12H  ITERATIONS)
      GO TO 20
15    WRITE (6,5) KS,I
5     FORMAT (I3,38HSINGULAR SET OF EQUATIONS ON ITERATION,I3)
20    CALL POWER(P,Q,ET,DT,Y,T,N)
      WRITE (6,22) (P(K),Q(K),ET(K),DT(K), K = 1,N)
22    FORMAT(//1X,3HPT=,11X,3HQT=,12X,3HET=,11X,3HDT=/(1X,4F11.5))
      CALL EXIT
      END
```

READ INPUT DATA

CALCULATE
$DP = P - P_{SKED}$
$DQ = Q - Q_{SKED}$

CHECK FOR CONVERGENCE

FORM
$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \underline{J} \cdot \Delta \underline{X}$$

SOLVE FOR ΔX

ADD CORRECTION FACTOR
$X_{i+1} = \Delta X + X_i$

WRITE P_T, Q_T
$|E|_T$, δ_T

Fig. D-2

Load Flow Program Listings

ESTIMATOR

The main program for the estimator is set-up to estimate the state from voltmeters wattmeters at buses, and varmeters at buses. The number of buses is arbitrary. The program allows the following variables to be changed on every run:

1) IE (j): a vector of dimension N E indicating the buses where voltmeters are placed.

2) I P (j): a vector of dimension NP indicating the buses where wattmeters are placed.

3) IQ (j): a vector of dimension NQ indicating the buses where varmeters are placed.

4) M M: the maximum number of iterations allowed for the estimator recursion.

5) EPS: the level of the convergence criterion. To converge $|\underline{x}_{i+1} - \underline{x}_i| \leq$ EPS. This is a fairly strong convergence criterion.

6) NM: the number of times an estimate is to be calculated.

7) $\sqrt{R}$: the standard deviations of the noise error convariance matrix.

The program prints out the following quantities:

1) $\underline{D}x = \underline{x}_T - \underline{x}$

2) $\Delta \underline{P} = \underline{P}_T - P(\underline{x})$

$\Delta \underline{Q} = \underline{Q}_T - Q(\underline{x})$

3) Diagonal elements of the matrix $\sum(\hat{\underline{x}})$.

See Fig. (D-3) for a flow chart and Fig. (D-4) for a listing.

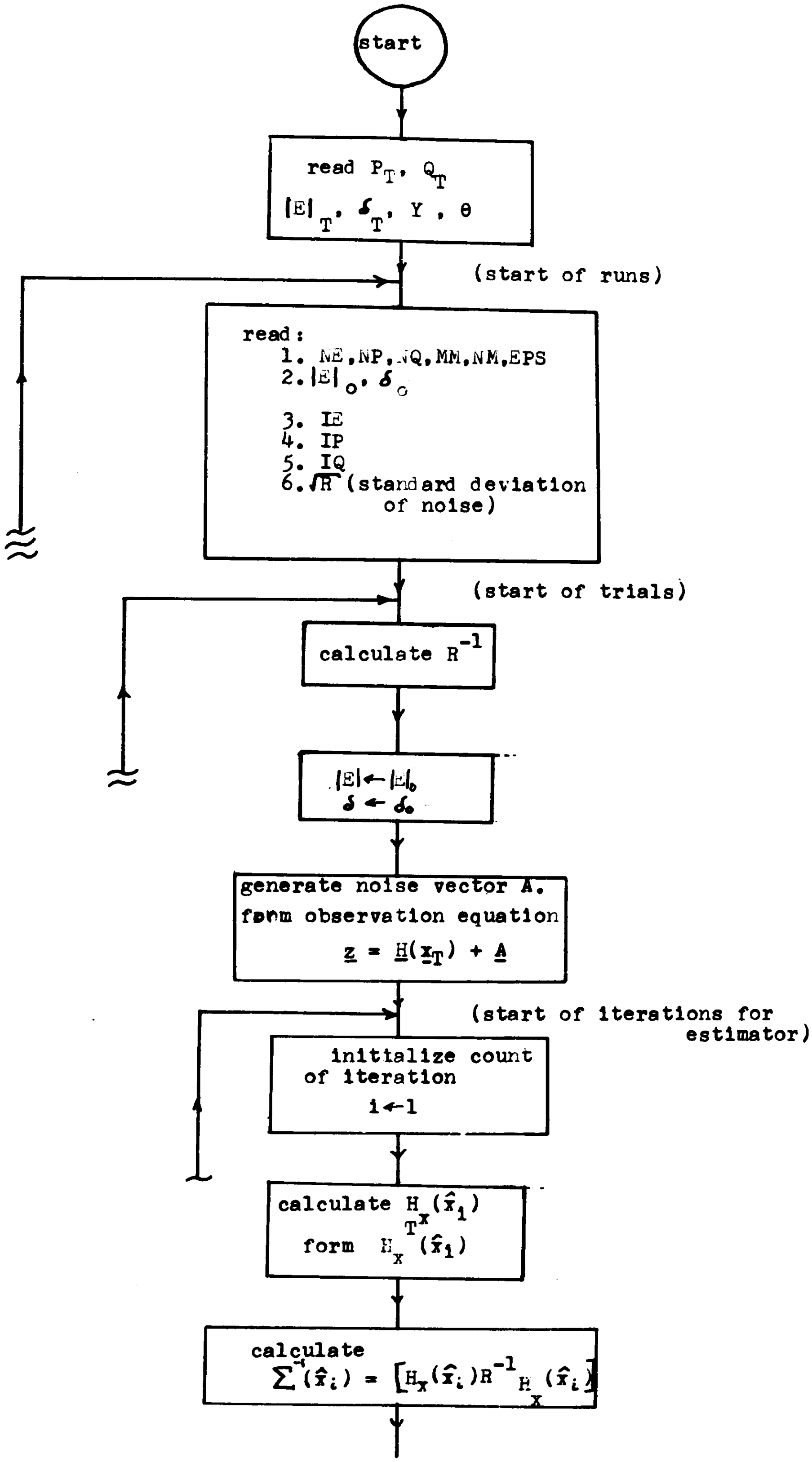

Fig. D-3

Flow Chart for Estimator

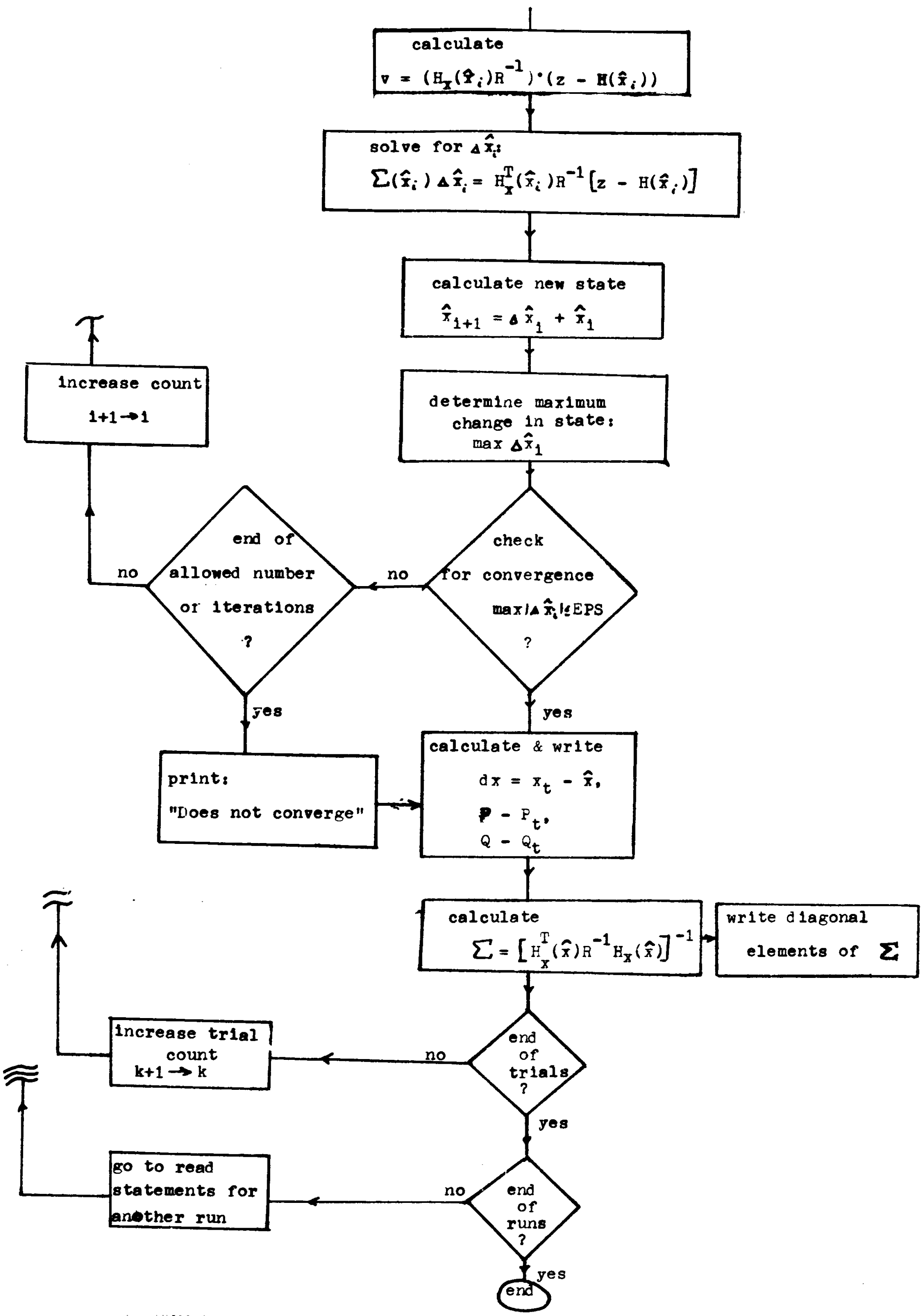

Fig. D-3
(continued)

```
FORTRAN IV G LEVEL 1, MOD 2          MAIN          DATE = 68207          20/19/01          PAGE 0001

      C     THIS IS THE MAIN PROGRAM FOR THE ESTIMATOR                          00010
      C                                                                         00020
            READ (5,1) N                                                        00021
            N2 = N*2-1                                                          00022
            N1 = N - 1                                                          00023
            N3 = N*3
            NBU = 1                                                             00024
            REAL J1,J2,J3 , J4,J,MAX                                            00030
            DIMENSION IE(5),IP(5),IQ(5),ZT(15),ET(5),DT(5),S(9,9),DX(9)         00040
           1 ,P(5),Q(5),Z(15),E(5),D(5),R(15),RI(15,15),SIGMA(9,9)              00050
           2 ,Y(5,5),T(5,5),J1(5,5),J2(5,5),J3(5,5),J4(5,5),EO(5),DO(5)         00060
           3 ,J(10,10),HX(15,15),HXT(15,15),DZ(15),V(9), HXT1(15,15),ISW(6)     00061
           4 ,PT(5),QT(5),LW(9),MW(9),DP(5),DQ(5)                               00071
       98   READ (5,4) (PT(K),QT(K),ET(K),DT(K),K=1,N)                          00232
       99   READ (5,4) ((Y(I,K),T(I,K),K=1,N),I=1,N)                            00234
            WRITE(6,42) (ET(K),DT(K),PT(K),QT(K),K=1,N)
       42    FORMAT(1H1//1X,4H ET=,10X,4H DT=,10X,3HPT=,10X,3HQT=/(1X,4F11.5))
      C
      100   READ (5,1) NSW, (ISW(K),K=1,NSW)                                    00080
            DO 107 I=1,NSW                                                      00090
            NGT = ISW(I)                                                        00100
            GO TO (101,102,103,104,105,106), NGT                                00110
      101   READ (5,2) NE,NP,NQ,MM,NM,EPS                                       00120
            WRITE (6,40) NM                                                     00121
       40   FORMAT(//2X,19HNUMBER OF TRIALS = ,I3)                              00122
            NT = NE + NP + NQ                                                   00130
            GO TO 107                                                           00140
      102   READ (5,3) (EO(K),DO(K),K=1,N)    ,PCI                              00150
            WRITE (6,41) PCI                                                    00151
       41   FORMAT(//1X,F5.1,32H PERCENT ERROR IN INITIAL VALUES)               00152
            GO TO 107                                                           00160
      103   READ (5,2) (IE(K),K=1,NE)                                           00170
            WRITE (6,202)                                                       00211
      202   FORMAT(//1X,18HOBSERVATION BUSSES)                                  00212
            WRITE(6,203) (IE(K),K=1,NE)                                         00213
      203   FORMAT(//1X,2HE:,5I3)                                               00214
            GO TO 107                                                           00180
      104   READ (5,2) (IP(K),K= 1,NP)                                          00190
            WRITE (6,204) (IP(K),K=1,NP)                                        00215
      204   FORMAT(//1X,2HP:,5I3)                                               00216
            GO TO 107                                                           00200
      105   READ (5,2) (IQ(K),K= 1,NQ)                                          00210
            WRITE(6,205) (IQ(K),K=1,NQ)                                         00217
      205   FORMAT(//1X,2HQ:,5I3)                                               00218
            CALL ZTRUE(IE,IP,IQ,NE,NP,NQ,ZT,ET,PT,QT,N,N3)
            WRITE(6,43) (ZT(K),K=1,NT)
       43   FORMAT (//1X,4H ZT=/(1X,F10.5))
            GO TO 107                                                           00220
```

READ DATA

Fig. D-4
Estimator Program Listing

```
FORTRAN IV G LEVEL 1, MOD 2             MAIN             DATE = 68207          20/19/01                 PAGE 0002

 0042          106    READ (5,3) (R(K),K= 1,NT), PC1,PC2,PC3                               00230
 0043                 WRITE (6,108) PC1,PC2,PC3                                            00231
 0044          108    FORMAT(//1X,33HPERCENT ERROR IN OBSERVATIONS: E=,F5.1,4H ,P=,        00232
                    1 F5.1,4H ,Q=,F5.1)                                                    00300
 0045                  IR = 1000.0*(R(3)+ R(5)+R(N2))
 0046          107    CONTINUE                                                             00240
 0047          2      FORMAT(1X,5I3,F10.5)                                                 00250
 0048          3      FORMAT (1X,5F11.5)                                                   00260
 0049          1      FORMAT (1X,7I3)                                                      00270
 0050          4      FORMAT(1X,4F11.5)                                                    00300
               C
               C      R INVERSE IS CALCULATED                                              00350
               C      S IS SET TO ZERO INITIALLY                                           00360
               C
 0051                 DO 113 I = 1,N2
 0052                 DO 113 K = 1,N2
 0053                 S(I,K) = 0.0                                                         00410
 0054          113    CONTINUE
 0055                 DO 13 I = 1,NT                                                       00380
 0056                 DO 14 K = 1,NT                                                       00390
 0057                 RI(I,K) = 0.0                                                        00400
 0058          14     CONTINUE                                                             00420
 0059                 RI(I,I) = 1.0/(R(I)**2)                                              00430
 0060          13     CONTINUE                                                             00440
               C
               C      START OF TRIALS TO CALCULATE S
               C
 0061                 DO 50 KK = 1,NM                                                      00450
 0062                 DO 45 K = 1,N                                                        00460
 0063                 E(K)= EO(K)                                                          00470
 0064                 D(K) = DO(K)                                                         00480
 0065          45      CONTINUE                                                            00490
               C
               C      OBSERVATION NOISE IS CALCULATED                                      00500
               C
 0066                 CALL ZTRUE(IE,IP,IQ,NE,NP,NQ,ZT,ET,PT,QT,N,N3)
 0067                 IX = 200*KK*NM*NBU*IR + 1                                            00510
 0068                 DO 10 K =1,NT                                                        00520
 0069                 B = R(K)                                                             00530
 0070                 CALL GAUSS(IX,B,0.,A)                                                00550
 0071                 Z(K) = A + ZT(K)                                                     00560
 0072          10     CONTINUE                                                             00570
               C
               C      START OF ITERATION                                                   00590
               C
 0073                 DO 11 L = 1,MM                                                       00600
 0074                 CALL POWER(P,Q,E,D,Y,T,N)                                            00740
 0075                 CALL ELEM(J1,J2,J3,J4,E,D,Y,T,P,Q,N)                                 00620
```

R^{-1} IS CALCULATED

START OF DO LOOP FOR TRIALS

NOISE GENERATED

$Z = H(X) + V$

START OF ITERATION

Fig. D-4
(continued)

```
              CALL HMATRX(IE,IP,IQ,NE,NP,NQ,J1,J2,J3,J4,HX,N,N3)
      C
      C       HX TRANSPOSE IS CALCULATED
              DO 12 I = 1,N2
              DO 12 K = 1,NT
              HXT(I,K) = HX(K,I)
         12   CONTINUE
      C
      C       HXT1 = HXT*RI
              CALL PRODM (HXT,RI,HXT1,N2,NT,NT,N3,N3)
      C       SIGMA = HXT1*HX
              CALL PRODM (HXT1,HX,SIGMA,N2,NT,N2,N3,N2)
      C
      C       V = HXT*RI*(Z - ZT(E,D))
              CALL ZTRUE(IE,IP,IQ,NE,NP,NQ,ZT,E,P,Q,N,N3)
              DO 15 I = 1,N2
              SUM = 0.0
              DO 16 K = 1 ,NT
              V1 = HXT1(I,K)*(Z(K) - ZT(K))
              SUM = V1 + SUM
         16   CONTINUE
              V(I) = SUM
         15   CONTINUE
              CALL SIMQ(SIGMA,V,N2,KS,N2)
              IF(KS .GE. 1) GO TO 20
      C       DX = V, SO X(K+1) = X(K) + V
              DO 17 I = 1,N1
              D(I) = V(I) + D(I)
              K = I + N -1
              E(I) = V(K) + E(I)
         17   CONTINUE
              D(N) = 0.0
              E(N) = V(N2) + E(N)
              VMAX = MAX(V,N2)
              IF (VMAX .LE. EPS) GO TO 21
         11   CONTINUE
              WRITE (6,201)
        201   FORMAT(//1X,26HESTIMATE HAS NOT CONVERGED)
              GO TO 22
         20   WRITE(6,30) L
         30   FORMAT(/1X,39H SINGULAR SET OF EQUATIONS ON ITERATION,I2)
              GO TO 22
         21   WRITE(6,31) L
         31   FORMAT(/1X,13HCONVERGES IN ,I2,12H ITERATIONS )
         22   DO 51 I = 1,N1
              DX(I) = D(I) - DT(I)
              K = I + N -1
              DX(K) = E(I) - ET(I)
```

$\Sigma^{-1} = [H_x^T R^{-1} H_x]$

$H_x^T R^{-1} [z - H(\hat{x}_i)]$

SOLVE FOR $\Delta\hat{X}_i$

$\hat{X}_{i+1} = \Delta\hat{X}_i + \hat{X}_i$

CHECK FOR CONVERGEN

$DX = \hat{X} - X_T$

Fig. D-4
(continued)

```
51    CONTINUE
      DX(N2) = E(N) - ET(N)
      WRITE (6,6) DX
6     FORMAT (//1X,3HDX=/(1X, 9F10.5))
      CALL POWER(P,Q,E,D,Y,T,N)
      CALL VSUM(P,PT,DP,-1.0,N)
      CALL VSUM(Q,QT,DQ,-1.0,N)
      WRITE (6,206) (P(K),Q(K),DP(K),DQ(K),K=1,N)
206   FORMAT(//3X,2HP=,8X,2HQ=,7X,5HP-PT=,5X,5HQ-QT=/(1X,4F10.5))
      CALL PRODM (HXT1,HX,SIGMA,N2,NT,N2,N3,N2)
      CALL MINV(SIGMA,N2,D,LW,MW)
      WRITE(6,54) (SIGMA(I,I),I=1,N2)
54    FORMAT(//1X,14HSIGMA DIAGONAL/1X,5HANGLE,1P4E15.3//1X,4HMAG.,1P
     15E15.3//)
50    CONTINUE
      NBU = NBU + 1
      GO TO 100
      END
```

CALCULATE

P - PT, Q - QT

CALCULATE

$$\Sigma = \left[H_x^T(\hat{x})\, R^{-1} H_x(\hat{x})\right]^{-1}$$

Fig. D-4
(continued)

SUBROUTINES

The following list gives the names of all the subroutine programs, a short description of each, and where the listing is found.

Name	Description	Figure
1. VSUM	vector sum	D-5
2. PRODM	matrix product	D-5
3. MAX	finds maximum element	D-5
4. ELEM	forms the elements of the matrix $\frac{\partial \underline{H}(\underline{x})}{\partial \underline{x}}$	D-6
5. JACOBN	forms the Jacobian matrix for load flow calculation from the elements	D-7
6. POWER	Calculates real and reactive bus powers	D-8
7. $\underline{Z}$ TRUE*	forms the true observation	D-8
8. $\underline{H}$ MATRIX*	forms the matrix $\frac{\partial \underline{H}}{\partial \underline{x}}(\underline{x})$ for the estimator calculations	D-9
9. SIM Q	simultaneous equation solver	SSP
10. MINV	matrix inversion	SSP
11. GAUSS	Gaussian random number generator	SSP

* indicates that the subroutine is described in more detail.

(SSP - Scientific Subroutine package program contained on 360 computer)

Z TRUE

This subroutine calculates the true observation vector. IE, IP, and IQ are subscripted subscripts indicating the buses at which there are $|E|$ meters, wattmeters, and varmeters respectively.

As an example let $|E|$ meters be placed at buses 1, 3, and 5, watt meters at buses 1, 2, 3 and 4, and var meters at buses 2, 3, 4, 5. The subroutine would then form the vector:

$$\underline{z} = \begin{bmatrix} |E_1| \\ |E_3| \\ |E_5| \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \end{matrix}}\right\} NE \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} NP \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} NQ \end{matrix}$$

Where NE, NP, and NQ are the number of voltmeters, wattmeters, and varmeters respectively. A program listing is found on Figure (D-8).

H MATRX

This subroutine calculates the elements of the Jacobian matrix for the estimator calculations. IE, IP, and IQ are subscripted subscripts indicating the buses at which voltmeters, wattmeters, and varmeters are placed.

As an example let $|E|$ meters be placed at buses 1, 3 and 5, wattmeters at buses 1, 2, 3, and 4 and varmeters at buses 2, 3, 4, and 5. The Jacobian matrix calculated by this subroutine is

$$H_x = \left[\begin{array}{cccc:ccccc}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hdashline
\frac{\partial P_1}{\partial \delta_1} & \cdots & & \frac{\partial P_1}{\partial \delta_4} & \frac{\partial P_1}{\partial |E|_1} & \cdots & & & \frac{\partial P_1}{\partial |E|_5} \\
\vdots & & & \vdots & \vdots & & & & \vdots \\
\frac{\partial P_4}{\partial \delta_1} & \cdots & & \frac{\partial P_4}{\partial \delta_4} & \frac{\partial P_4}{\partial |E|_1} & \cdots & & & \frac{\partial P_4}{\partial |E|_5} \\
\hdashline
\frac{\partial Q_2}{\partial \delta_1} & \cdots & & \frac{\partial Q_2}{\partial \delta_4} & \frac{\partial Q_2}{\partial |E|_1} & \cdots & & & \frac{\partial Q_2}{\partial |E|_5} \\
\vdots & & & \vdots & \vdots & & & & \vdots \\
\frac{\partial Q_5}{\partial \delta_1} & \cdots & & \frac{\partial Q_5}{\partial \delta_4} & \frac{\partial Q_5}{\partial |E|_1} & \cdots & & & \frac{\partial Q_5}{\partial |E|_5}
\end{array}\right]$$

where NE, NP, and NQ are the number of voltmeters, wattmeters and varmeters respectively. Notice that only four angles are used in the partial derivatives. This is because only the differences in angles are important, so one angle serves as a reference. In the computer program δn is the reference angle and it is set to zero. A program listing is found in Figure (D-9).

```
FORTRAN IV G LEVEL 1, MOD 2              VSUM              DATE = 68207          20/19/01            PAGE 0001

 0001              SUBROUTINE VSUM(X,Y,Z,A,N)                                              00010
 0002              DIMENSION X(N), Y(N), Z(N)                                              00020
 0003              DO 10 I= 1,N                                                            00030
 0004              Z(I) = X(I) + A*Y(I)                                                    00040
 0005        10    CONTINUE                                                                00050
 0006              RETURN                                                                  00060
 0007              END                                                                     00070
```

VECTOR SUM

```
FORTRAN IV G LEVEL 1, MOD 2              PRODM             DATE = 68207          20/19/01            PAGE 0001

 0001              SUBROUTINE PRODM(A,B,C,N,M,L,KK,LL)                                     00010
 0002              DIMENSION A(KK,KK),B(KK,KK), C(LL,LL)
 0003              DO 10 I = 1,N                                                           00030
 0004              DO 10 J = 1,L                                                           00040
 0005              SUM = 0.0                                                               00050
 0006              DO 12 K= 1,M                                                            00060
 0007              SUM= A(I,K)*B(K,J) + SUM                                                00070
 0008        12    CONTINUE                                                                00080
 0009              C(I,J) = SUM                                                            00090
 0010        10    CONTINUE                                                                00100
 0011              RETURN                                                                  00110
 0012              END                                                                     00120
```

MATRIX PRODUCT

```
FORTRAN IV G LEVEL 1, MOD 2              MAIN              DATE = 68207          20/19/01            PAGE 0001

             C     THIS FUNCTION FINDS THE MAXIMUM VALUE OF A LIST                         00010
 0001              REAL FUNCTION MAX(Y,N)                                                  00020
 0002              DIMENSION Y(N)                                                          00030
 0003              TEMP = ABS(Y(1))                                                        00040
 0004              DO 10 I= 2,N                                                            00050
 0005              AY = ABS(Y(I))                                                          00060
 0006              IF (TEMP .GE. AY) GO TO 10                                              00070
 0007              TEMP = ABS(Y(I))                                                        00080
 0008        10    CONTINUE                                                                00090
 0009              MAX = TEMP                                                              00100
 0010              RETURN                                                                  00110
 0011              END                                                                     00120
```

Fig. D-5

Subprograms VSUM, PRODM and MAX

```
FORTRAN IV G LEVEL 1, MOD 2            MAIN            DATE = 68207        20/19/01             PAGE 0001

                C      THIS SUBROUTINE CALCULATES THE ELEMENTS OF
                C      THE JACOBIAN INCLUDING THOSE AT THE SLACK BUS (BUS N)
                C
   0001                SUBROUTINE ELEM(J1,J2,J3,J4,E,D,Y,T,P,Q,N)
   0002                REAL  J1,J2,J3,J4, J2D, J4D
   0003                DIMENSION J1(N,N),J2(N,N),J3(N,N),J4(N,N),E(N)
                      1 ,D(N),Y(N,N),T(N,N)
                C      J1 = (DP/DD.), J3 =(DQ/DD)
                C      J1 AND J3 ARE FIRST CALCULATED
   0004                DO 10 I = 1,N
   0005                DO 10 K = 1,N
   0006                F = E(I) * E(K) * Y(I,K)
   0007                J1(I,K)= F*SIN(T(I,K) + D(I) - D(K))
   0008                J3(I,K) = -F*COS(T(I,K) + D(I) - D(K))
   0009         10     CONTINUE
   0010                DIMENSION P(N), Q(N)
   0011                DO 11 I = 1,N
   0012                J1(I,I) = J1(I,I) - Q(I)
   0013                J3(I,I) = J3(I,I) + P(I)
   0014         11     CONTINUE
                C      J2 = (DP/DE) , J4 = (DQ/DE)
                C      J2 AND J4 ARE NOW CALCULATED
   0015                DO 12 I = 1,N
   0016                DO 12 K = 1,N
   0017                J2(I,K) = E(I)*Y(I,K)*COS(T(I,K) + D(I) - D(K))
   0018                J4(I,K) = E(I)*Y(I,K)*SIN(T(I,K) + D(I) - D(K))
   0019         12     CONTINUE
   0020                DO 13 I=1,N
   0021                SUM2 = 0.0
   0022                SUM4 = 0.0
   0023                DO 14 K = 1,N
   0024                J2D = E(K)*Y(I,K)*COS(T(I,K) + D(I) - D(K))
   0025                J4D = E(K)*Y(I,K)*SIN(T(I,K) + D(I) - D(K))
   0026                SUM2 = SUM2 + J2D
   0027                SUM4 = SUM4 + J4D
   0028         14     CONTINUE
   0029                J2(I,I) = J2(I,I) + SUM2
   0030                J4(I,I) = J4(I,I) + SUM4
   0031         13     CONTINUE
   0032                RETURN
   0033                END
```

$J1 = \frac{\partial P}{\partial \delta}$

$J2 = \frac{\partial Q}{\partial \delta}$

$J3 = \frac{\partial P}{\partial E}$

$J4 = \frac{\partial Q}{\partial E}$

Fig. D-6
Subroutine ELEM

```
C     THIS SUBROUTINE FORMS THE JACOBIAN

      SUBROUTINE JACOBN(J,E,D,Y,T,N,NN2)
      REALJ1,J2,J3,J4 , J
      DIMENSION J(NN2,NN2),J1(N,N),J2(N,N),J3(N,N),J4(N,N)
     1 ,E(N),D(N),Y(N,N),T(N,N)
      CALL ELEM (J1,J2, J3, J4, E, D, Y, T, N)
      M= N-1
      DO 10 I= 1,M
      DO 10 K= 1,M
      II = I+N-1
      KK = K+N-1
      J(I,K) = J1(I,K)
      J(II,K) = J3(I,K)
      J(I,KK) = J2(I,K)
      J(II,KK) = J4(I,K)
   10 CONTINUE
      RETURN
      END
```

Fig. D-7

Subroutine JACOBN

```
FORTRAN IV G LEVEL 1, MOD 2            MAIN           DATE = 68207           20/19/01                PAGE 0001

      C     THIS SUBROUTINE CALCULATES REAL POWER P(I)
      C     AND REACTIVE POWER Q(I)
            SUBROUTINE POWER(P,Q,E,D,Y,T,N)
            DIMENSION P(N),Q(N),E(N),D(N),Y(N,N),T(N,N)
            DO 10  I = 1,N
            SUMP = 0.0
            SUMQ = 0.0
            DO 11 J= 1,N
            REAL M
            M = E(I)*E(J)*Y(I,J)
            PJ = M*COS(T(I,J)+ D(I) - D(J))
            QJ = M*SIN(T(I,J) + D(I)- D(J))
            SUMP = PJ + SUMP
            SUMQ = QJ + SUMQ
       11   CONTINUE
            P(I) = SUMP
            Q(I) = SUMQ
       10   CONTINUE
            RETURN
            END
```

```
FORTRAN IV G LEVEL 1, MOD 2            MAIN           DATE = 68207           20/19/01                PAGE 0001

      C     THIS  SUBROUTINE CAL CULATES THE TRUE OBSERVATIONS ZT = H(X)
      C     IE,IP,IQ ARE SUBSCRIPTED SUBSCRIPTS
      C     INDICATING WHICH OBSERVATIONS ARE BEING USED
      C
            SUBROUTINE ZTRUE(IE,IP,IQ,NE,NP,NQ,ZT,E,P,Q,N,N3)
            DIMENSION IE(N),IP(N),IQ(N),ZT(N3),P(N),Q(N) ,E(N)
            IF (NE .EQ. 0) GO TO 20
            DO 10 J = 1,NE
            IJ = IE(J)
            ZT(J) = E(IJ)
       10    CONTINUE
       20   IF (NP .EQ. 0) GO TO 21
            DO 11 L = 1,NP
            KL = IP(L)
            LL = L + NE
            ZT(LL) = P(KL)
       11   CONTINUE
       21   IF (NQ .EQ. 0) GO TO 22
            DO 12 M = 1,NQ
            MN = IQ(M)
            NN = M + NP + NE
            ZT(NN) = Q(MN)
       12   CONTINUE
       22   RETURN
            END
```

Fig. D-8

Subroutines POWER and Z TRUE

```
FORTRAN IV G LEVEL 1, MOD 2            MAIN           DATE = 68207         20/19/01            PAGE 0001

      C      THIS SUBROUTINE CALCULATES THE MATRIX DH/DX
      C      WHICH IS WRITTEN AS HX.  THE VECTORS
      C      IE,IP,IQ ARE SUBSCRIPTED SUBSCRIPTS
      C      INDICATING WHICH OBSERVATIONS ARE USED
      C
             SUBROUTINE HMATRX(IE,IP,IQ,NE,NP,NQ,J1,J2,J3,J4,HX,N,N3)
             REAL J1,J2,J3 , J4
             DIMENSION IE(N), IP(N),IQ(N),J1(N,N),J2(N,N),J3(N,N),
            1 J4(N,N),HX(N3,N3)
             N2 = N*2 - 1
             N1 = N-1
             IF (NE .EQ. 0) GO TO 20
             DO 10 I = 1,NE
             DO 10 J = 1,N2
             HX(I,J) = 0.0
       10    CONTINUE
             DO 11 J = 1,NE
             IJ = IE(J) + N - 1
             HX(J,IJ) = 1.0
       11    CONTINUE
       20    IF (NP .EQ. 0) GO TO 21
             DO 12 L = 1,NP
             KL = IP(L)
             LL = NE + L
             DO 14 I = 1,N1
             II = I + N - 1
             HX(LL,I) = J1(KL,I)
             HX(LL,II) = J2(KL,I)
       14    CONTINUE
             HX(LL,N2) = J2(KL,N)
       12    CONTINUE
       21    IF (NQ .EQ. 0) GO TO 22
             DO 13 L = 1,NQ
             MN = IQ(L)
             NN = NE + NP + L
             DO 15 M = 1,N1
             HX(NN,M) = J3(MN,M)
             MM = N + M - 1
             HX(NN,MM) = J4(MN,M)
       15    CONTINUE
             HX(NN,N2) = J4(MN,N)
       13    CONTINUE
       22    RETURN
             END
```

Fig. D-9

Subroutine H MATRIX

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